

QQQ – Further Statistics 1 – Chapter 2 – Poisson Distributions (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. The discrete random variables W , X and Y are distributed as follows

$$W \sim B(10, 0.4)$$

$$X \sim \text{Po}(4)$$

$$Y \sim \text{Po}(3)$$

(a) Explain whether or not $\text{Po}(4)$ would be a good approximation to $B(10, 0.4)$ (1)

(b) State the assumption required for $X + Y$ to be distributed as $\text{Po}(7)$ (1)

Given the assumption in part (b) holds,

(c) find $P(X + Y < \text{Var}(W))$ (2)

(Total 4 marks)

Edexcel A2 October 2020 Q2

2. Indre works on reception in an office and deals with all the telephone calls that arrive. Calls arrive randomly and, in a 4-hour morning shift, there are on average 80 calls.

(a) Using a suitable model, find the probability of more than 4 calls arriving in a particular 20-minute period one morning. (3)

Indre is allowed 20 minutes of break time during each 4-hour morning shift, which she can take in 5-minute periods. When she takes a break, a machine records details of any call in the office that Indre has missed.

One morning Indre took her break time in 4 periods of 5 minutes each.

(b) Find the probability that in exactly 3 of these periods there were no calls. (2)

On another occasion Indre took 1 break of 5 minutes and 1 break of 15 minutes.

(c) Find the probability that Indre missed exactly 1 call in each of these 2 breaks. (3)

(Total 8 marks)

Edexcel A2 June 2019 Q2

3.

- (a) State two conditions under which a Poisson distribution is a suitable model to use in statistical work. (2)

The number of cars passing an observation point in a 10 minute interval is modelled by a Poisson distribution with mean 1.

- (b) Find the probability that in a randomly chosen 60 minute period there will be
- (i) exactly 4 cars passing the observation point,
 - (ii) at least 5 cars passing the observation point. (5)

The number of other vehicles, other than cars, passing the observation point in a 60 minute interval is modelled by a Poisson distribution with mean 12.

- (c) Find the probability that exactly 1 vehicle, of any type, passes the observation point in a 10 minute period. (4)

(Total 11 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
requires large n /small p so not a good approximation	B1	3.5b
	(1)	

Notes

B1: Correct reason why the model would not be appropriate and correct conclusion. Condone e.g. ' p is close to 0.5' for p is not small.

Mean is not equal to variance on its own in B0.

Mark scheme (b)

Scheme	Marks	AO
X and Y must be independent	B1	2.4
	(1)	

Notes

B1: Correct explanation mentioning independence (oe).

Ignore extraneous comments.

Mark scheme (c)

Scheme	Marks	AO
$P(X + Y < 2.4)$ from $Po(7)$ [$P(X + Y \leq 2)$]	M1	3.4
$= 0.029636 \dots$ awrt <u>0.0296</u>	A1	1.1b
	(2)	
	(4 marks)	

Notes

M1: Using $Po(7)$ with 2.4

A1: awrt 0.0296

2.

Mark scheme (a)

Scheme	Marks	AO
{Let C = no of calls in a 20 min period} $C \sim \text{Po}(\dots)$	M1	3.3
80 calls per 4-hour period gives $\frac{20}{3}$ per 20 mins i.e. $C \sim \text{Po}\left(\frac{20}{3}\right)$ $[P(C > 4)] = 1 - P(C \leq 4)$	M1	3.4
$= 0.79437\dots$ awrt <u>0.794</u>	A1	1.1b
	(3)	

Notes

1st M1 for selecting a Poisson model – written or used. May be implied by 2nd M1 or a correct Answer.

2nd M1 for the correct Poisson $\text{Po}\left(\frac{20}{3}\right)$ or $\text{Po}(6.67)$ or better seen and writing or using $1 - P(C \leq 4)$

A1 for awrt 0.794 (correct ans with no incorrect working scores 3/3)

Mark scheme (b)

Scheme	Marks	AO
{ X = no. of 5 min periods with no calls} $X \sim B\left(4, e^{-\frac{5}{3}}\right)$	M1	3.3
$P(X = 3) = 0.02186125\dots$ awrt <u>0.0219</u>	A1	1.1b
	(2)	

Notes

M1 for selecting a correct model $B(4, 0.189)$ or better (calc: 0.188875...)

A1 for using the model to get awrt 0.0219 (correct ans with no incorrect working scores 2/2)

Mark scheme (c)

Scheme	Marks	AO
$P(\text{exactly one call}) = e^{-\frac{5}{3}} \times \frac{5}{3}$ or $e^{-5} \times 5$	M1	2.1

$P(\text{exactly one call in each break}) = \left(e^{-\frac{5}{3}} \times \frac{5}{3}\right) \times (e^{-5} \times 5)$	M1	1.1b
$= 0.0106052 \dots$ awrt 0.0106	A1	1.1b
	(3)	
	(8 marks)	

Notes

1st M1 for a correct prob of 1 call (expressions in e or values)

(allow 0.31479... or awrt 0.315 or 0.033689... or awrt 0.0337)

2nd M1 for a correct probability statement or expression.

E.g. $P(S = 1 \mid S \sim \text{Po}(\frac{5}{3})) \times P(T = 1 \mid T \sim \text{Po}(5))$

SC e.g. $F \sim \text{Po}(\lambda)$ used in (b) to find $P(F = 0)$

Then if we see $Y \sim \text{Po}(3\lambda)$ and statement $P(F = 1) \times P(Y = 1)$ award M0M1

A1 for awrt 0.0106 (correct ans with no incorrect working scores 3/3)

3.

Mark scheme (a)

Scheme	Marks
<u>Events</u> occur at a constant rate. <u>Events</u> occur independently or randomly. <u>Events</u> occur singly.	B1 B1
	(2)

Notes

any two of the 3

B1 B1 Need the word events at least once.

Independently and randomly are the same reason.

Award the first B1 if they only gain 1 mark

Special case. If they have 2 of the 3 lines without the word events they get B0 B1

Mark scheme (b)

Scheme	Marks
--------	-------

Let X be the random variable the number of cars passing the observation point. $Po(6)$	B1
(i) $P(X \leq 4) - P(X \leq 3) = 0.2851 - 0.1512$ or $\frac{e^{-6}6^4}{4!}$	M1
$= 0.1339$	A1
(ii) $1 - P(X \leq 4) = 1 - 0.2851$ or $1 - e^{-6} \left(\frac{6^4}{4!} + \frac{6^3}{3!} + \frac{6^2}{2!} + \frac{6}{1!} + 1 \right)$	M1
$= 0.7149$	A1
	(5)

Notes

B1 Using $Po(6)$ in (i) or (ii)

(i) **M1** Attempting to find $P(X \leq 4) - P(X \leq 3)$ or $\frac{e^{-\lambda}\lambda^4}{4!}$

A1 awrt 0.134

(ii) **M1** Attempting to find $1 - P(X \leq 4)$

A1 awrt 0.715

Mark scheme (c)

Scheme	Marks
$P(0 \text{ car and } 1 \text{ others}) + P(1 \text{ cars and } 0 \text{ other})$	B1
$= e^{-1} \times 2e^{-2} + 1e^{-1} \times e^{-2}$ $= 0.3679 \times 0.2707 + 0.3679 \times 0.1353$	M1 A1
$= 0.0996 + 0.0498$ $= 0.149$	A1
	(4)
	(11 marks)

Notes

B1 Attempting to find both possibilities. May be implied by doing $e^{-\lambda_1} \times \lambda_2 e^{-\lambda_2} + e^{-\lambda_2} \times \lambda_1 e^{-\lambda_1}$ any values of λ_1 and λ_2

M1 finding one pair of form $e^{-\lambda_1} \times \lambda_2 e^{-\lambda_2}$ any values of λ_1 and λ_2

A1 one pair correct

A1 awrt 0.149

(corrected from the printed mark scheme: the second product is printed as 0.3674×0.1353 ;
 $e^{-1} = 0.3679$)

alternative

Scheme	Marks
$P_o(1 + 2) = P_o(3)$	B1
$P(X = 1) = 3e^{-3}$	M1 A1
$= 0.149$	A1

B1 for Po(3)

M1 for attempting to find $P(X = 1)$ with Po(3)

A1 $3e^{-3}$

A1 awrt 0.149