

QQQ – Further Statistics 1 – Chapter 1 – Discrete Random Variables (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. The discrete random variable X has probability distribution,

x	-1	0	1	3	7
$P(X = x)$	p	r	p	0.3	r

where p and r are probabilities.

Given that $E(X) = 1.95$

find the exact value of $E(\sqrt{X+1})$ giving your answer in the form $a + b\sqrt{2}$ where a and b are rational. (6)

(Total 6 marks)

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2. The discrete random variable X has probability distribution

x	-2	-1	0	1	3
$P(X = x)$	0.25	a	b	a	0.30

where a and b are probabilities.

(a) Find $E(X)$ (2)

Given that $\text{Var}(X) = 3.9$

(b) find the value of a and the value of b (4)

The independent random variables X_1 and X_2 each have the same distribution as X

(c) Find $P(X_1 + X_2 > 3)$ (3)

(Total 9 marks)

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3. The discrete random variable X has probability distribution

x	-1	a	b
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

where a and b are positive constants.

(a) Find an expression for $E(X)$ in terms of a and b **(2)**

The discrete random variable Y is defined as $Y = a + bX$

Given that $\text{Var}(Y) = \frac{1}{4}\text{Var}(X)$ and $E(Y) = \frac{5}{16}$

(b) find the value of $E(X)$ **(7)**

(Total 9 marks)

Solutions

1.

Scheme	Marks	AO												
$[E(X) =] -1 \times p + 0 + 1 \times p + 3 \times 0.3 + 7r$ <u>or</u> $0.9 + 7r$ [$= 1.95$]	M1	3.1a												
$[7r = 1.05 \text{ so}]$ <u>$r = 0.15$</u>	A1	1.1b												
[Sum of probs = 1] $2p + 0.3 + 2r = 1$ (o.e.)	M1	2.1												
<u>$p = 0.2$</u>	A1ft	1.1b												
[Let $Y = \sqrt{X + 1}$] <table><tr><td>y</td><td>0</td><td>1</td><td>$\sqrt{2}$</td><td>2</td><td>$\sqrt{8}$ or $2\sqrt{2}$</td></tr><tr><td>$P(Y = y)$</td><td>“0.2”</td><td>“0.15”</td><td>“0.2”</td><td>0.3</td><td>“0.15”</td></tr></table>	y	0	1	$\sqrt{2}$	2	$\sqrt{8}$ or $2\sqrt{2}$	$P(Y = y)$	“0.2”	“0.15”	“0.2”	0.3	“0.15”		
y	0	1	$\sqrt{2}$	2	$\sqrt{8}$ or $2\sqrt{2}$									
$P(Y = y)$	“0.2”	“0.15”	“0.2”	0.3	“0.15”									
$[E(Y) =] 0 + \text{‘0.15’} + \text{‘0.2’}\sqrt{2} + 2 \times 0.3 + \text{‘0.15’} \times \sqrt{8}$	M1	3.1a												
$=$ <u>$0.75 + 0.5\sqrt{2}$</u>	Alcao	1.1b												
	(6 marks)													

Notes

1st M1 for an attempt at an expression for $E(X)$ at least 3 correct non-zero products

1st A1 for $r = 0.15$ oe

2nd M1 for using sum of probabilities = 1 to form an equation for p (ft their value or letter r)

2nd A1ft for $p = 0.2$ oe or ft their r for $p = \frac{0.7 - 2 \times "0.15"}{2}$ (provided p is a probability)

3rd M1 for attempt at $E(Y)$ with at least 2 correct y values and products ft their p and r

3rd A1 cao allow fraction or decimal, also allow $\frac{3 + 2\sqrt{2}}{4}$

2.

Mark scheme (a)

Scheme	Marks	AO
$[E(X) =] -2 \times 0.25 + -1 \times a + 0 \times b + 1 \times a + 3 \times 0.3$	M1	1.1b
$= \underline{0.4}$	A1	1.1b
	(2)	

Notes

M1 for a correct attempt (at least 3 correct non-zero terms or products and addition)

division by k ($k \neq 1$) is M0

A1 for 0.4 o.e. (correct answer only scores 2 out of 2)

Mark scheme (b)

Scheme	Marks	AO
$E(X^2) = (-2)^2 \times 0.25 + (-1)^2 \times a + 0 + 1^2 \times a + 3^2 \times 0.3 (= 2a + 3.7)$	M1	2.1
$[Var(X) =] 3.9 = 2a + 3.7 - "0.4^2"$	dM1	1.1b
$a = \underline{0.18}$	A1	1.1b
[Use of sum of probs = 1 implies $2a + b = 0.45$] $b = \underline{0.09}$	A1ft	1.1b
	(4)	

Notes

1st M1 for a correct attempt at $E(X^2)$ (at least 3 correct non-zero products and addition)

Missing brackets around -2 and -1 is M0 unless recovered

2nd dM1 (dep on 1st M1) for use of $3.9 = \text{their } E(X^2) - [E(X)]^2$ ft their $E(X) = 0.4$

1st A1 for $a = 0.18$ o.e.

2nd A1 (dep on 1st M1 only) for $b = 0.09$ o.e. or their $b = 0.45 - 2 \times "a"$ (provided both a and b are probabilities)

Mark scheme (c)

Scheme	Marks	AO
$X_1 + X_2 > 3$ when $X_1 = 3, X_2 = 1 \quad X_1 = 1, X_2 = 3 \quad X_1 = 3, X_2 = 3$	M1	3.4

$[P(X_1 + X_2 > 3) =]$ $"0.18" \times 0.3 + 0.3 \times "0.18" + 0.3 \times 0.3$ or $2 \times 0.3 \times (0.3 + "0.18") - 0.3^2$	M1	1.1b
$= \underline{0.198}$	A1	1.1b
	(3)	
	(9 marks)	

Notes

1st M1 for identifying at least 2 cases e.g. $X_1 = 3, X_2 \geq 1$ counts as 2 cases

(ignore extras including any incorrect pairs identified)

implied by at least two correct products of probs. or correct ft products of probs.

2nd M1 for a correct numerical expression for the probability ft their "0.18"

A1 for 0.198 o.e.

3.

Mark scheme (a)

Scheme	Marks	AO
$E(X) = -1 \times \frac{1}{2} + \frac{1}{4} \times a + \frac{1}{4} \times b$	M1	1.1b
$[E(X) =] \frac{1}{4}a + \frac{1}{4}b - \frac{1}{2}$	A1	1.1b
	(2)	

Notes

M1: Attempt at $E(X)$, at least two terms correct

A1: Full correct expression for $E(X)$, may be unsimplified

Mark scheme (b)

Scheme	Marks	AO
$\text{Var}(Y) = b^2 \text{Var}(X)$	M1	1.1b
$b^2 = \frac{1}{4} \rightarrow b = \frac{1}{2}$	A1	1.1b

$E(Y) = a + bE(X)$ or <table border="1"><tr><td>$a - b$</td><td>$a + ab$</td><td>$a + b^2$</td></tr><tr><td>0.5</td><td>0.25</td><td>0.25</td></tr></table>	$a - b$	$a + ab$	$a + b^2$	0.5	0.25	0.25	B1	2.1
$a - b$	$a + ab$	$a + b^2$						
0.5	0.25	0.25						
$a + b \left(\frac{1}{4}a + \frac{1}{4}b - \frac{1}{2} \right) = \frac{5}{16}$ or $\frac{1}{2}(a - b) + \frac{1}{4}(a + ab) + \frac{1}{4}(a + b^2) = \frac{5}{16}$	M1	1.1b						
$a = \frac{4}{9}$	A1	1.1b						
$E(X) = \frac{1}{4} \left(\frac{4}{9} \right) + \frac{1}{4} \left(\frac{1}{2} \right) - \frac{1}{2}$	M1	3.1a						
$E(X) = -\frac{19}{72}$	A1	1.1b						
	(7)							
	(9 marks)							

Notes

M1: Writing or using $\text{Var}(Y) = b^2 \text{Var}(X)$

A1: For finding $b = \frac{1}{2}$

B1: Writing using $E(Y) = a + bE(X)$, or writing a correct probability distribution for Y

M1: Correct expression for $E(Y)$ in terms of a and b (or their value of b)

and equating their expression to $\frac{5}{16}$

A1: $a = \frac{4}{9}$ or exact equivalent

M1: Substituting their values of a and b into their expression for $E(X)$ where $a, b \neq \frac{5}{16}$

A1: $E(X) = -\frac{19}{72}$ or exact equivalent