

QQQ – Further Statistics 1 – Chapter 1 – Discrete Random Variables (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. The probability distribution of the discrete random variable X is

$$P(X = x) = \begin{cases} \frac{k}{x} & \text{for } x = 1, 2 \text{ and } 3 \\ \frac{m}{2x} & \text{for } x = 6 \text{ and } 9 \\ 0 & \text{otherwise} \end{cases}$$

where k and m are positive constants.

Given that $E(X) = 3.8$, find $\text{Var}(X)$

(7)

(Total 7 marks)

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2. The discrete random variable X has the following probability distribution.

x	-5	-2	3	4
$P(X = x)$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$

- (a) Find $\text{Var}(X)$

(3)

The discrete random variable Y is defined in terms of the discrete random variable X

When X is negative, $Y = X^2$

When X is positive, $Y = 3X - 2$

- (b) Find $P(Y < 9)$

(3)

- (c) Find $E(XY)$

(2)

(Total 8 marks)

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3. The discrete random variable X has probability distribution

x	-5	-1	0	5	b
$P(X = x)$	0.3	0.25	0.1	0.15	0.2

where b is a constant and $b > 5$

(a) Find $E(X)$ in terms of b **(1)**

Given that $\text{Var}(X) = 34.26$

(b) find the value of b **(4)**

(c) Find $P(X^2 < 2 - 3X)$ **(4)**

(Total 9 marks)

Solutions

1.

Scheme	Marks	AO
$\Sigma p = 1 \rightarrow k + \frac{k}{2} + \frac{k}{3} + \frac{m}{12} + \frac{m}{18} = 1$ $\Sigma px = 3.8 \rightarrow k + \frac{k}{2}(2) + \frac{k}{3}(3) + \frac{m}{12}(6) + \frac{m}{18}(9) = 3.8$	M1	3.1a
$\frac{11k}{6} + \frac{5m}{36} = 1 \quad [= 66k + 5m = 36]$	A1	1.1b
$3k + m = 3.8$	A1	1.1b
Solving simultaneously to eliminate one variable	dM1	1.1b
$k = \frac{1}{3}$ and $m = \frac{14}{5}$	A1	1.1b
$E(X^2) = 1^2 \times k + 2^2 \times \frac{k}{2} + 3^2 \times \frac{k}{3} + 6^2 \times \frac{m}{12} + 9^2 \times \frac{m}{18} \quad [= 23]$	M1	1.1b
$\text{Var}(X) = 23 - 3.8^2$		
$= 8.56$	A1	1.1b
	(7 marks)	

Notes

M1: Attempt at both required equations with at least one term in k and one term in m correct

A1: Correct equation using $\Sigma p = 1$

A1: Correct equation using $\Sigma px = 3.8$

dM1: (dep on 1st M1) Solving simultaneously (may be implied by one correct value found)

A1: both values correct (may be implied by correct answer)

M1: Attempt to find $E(X^2)$ using their value of k and their value of m with at least 3 correct products or correct ft products Note: $E(X^2) = 6k + 7.5m$

A1: 8.56 cao

2.

Mark scheme (a)

Scheme	Marks	AO
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$[E(X) =] (-5) \times \frac{1}{12} + (-2) \times \frac{1}{6} + (3) \times \frac{1}{4} + (4) \times \frac{1}{2} [= 2]$	M1	1.1b
$[E(X^2) =]$ $(-5)^2 \times \frac{1}{12} + (-2)^2 \times \frac{1}{6} + (3)^2 \times \frac{1}{4} + (4)^2 \times \frac{1}{2} [= 13] \text{ (oe)}$	M1	1.1b
$\text{Var}(X) = E(X^2) - [E(X)]^2 = 13 - 2^2 = \underline{9}$	A1	1.1b
	(3)	

Notes

M1: Attempt at $E(X)$ with at least 3 correct products seen

M1: Attempt at $E(X^2)$ with at least 3 correct products seen

A1: 9 cao

Alternative

M1: Attempt at $E(X)$ with at least 3 correct products seen

M1: Attempt at expression for

$E((X - \mu)^2) = (-5 - 2)^2 \times \frac{1}{12} + (-2 - 2)^2 \times \frac{1}{6} + (3 - 2)^2 \times \frac{1}{4} + (4 - 2)^2 \times \frac{1}{2}$ with at least 3 correct terms

A1: 9 cao

Mark scheme (b)

Scheme	Marks	AO															
<table><tr><td>x</td><td>(-5)</td><td>-2</td><td>3</td><td>(4)</td></tr><tr><td>y</td><td>(25)</td><td>4</td><td>7</td><td>(10)</td></tr><tr><td>p</td><td>$(\frac{1}{12})$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{4}$</td><td>$(\frac{1}{2})$</td></tr></table>	x	(-5)	-2	3	(4)	y	(25)	4	7	(10)	p	$(\frac{1}{12})$	$\frac{1}{6}$	$\frac{1}{4}$	$(\frac{1}{2})$	M1	3.1a
x	(-5)	-2	3	(4)													
y	(25)	4	7	(10)													
p	$(\frac{1}{12})$	$\frac{1}{6}$	$\frac{1}{4}$	$(\frac{1}{2})$													
$P(Y < 9) = P(X = -2) + P(X = 3) \left[= \frac{1}{6} + \frac{1}{4} \right]$	M1	1.1b															
$= \underline{\underline{\frac{5}{12}}}$	A1	1.1b															
	(3)																

Notes

M1: Finding distribution of Y

M1: $P(X = -2) + P(X = 3)$ or $P(Y = 4) + P(Y = 7)$

A1: $\frac{5}{12}$ (condone awrt 0.417)

Mark scheme (c)

Scheme	Marks	AO
$E(XY) = (-5)(25)\frac{1}{12} + (-2)(4) \times \frac{1}{6} + (3)(7) \times \frac{1}{4} + (4)(10) \times \frac{1}{2}$	M1	3.1a
$= \underline{13.5}$	A1	1.1b
	(2)	
	(8 marks)	

Notes

M1: Attempt at $E(XY)$ with at least 2 correct terms

A1: 13.5

3.

Mark scheme (a)

Scheme	Marks	AO
$[E(X) =] 0.2b - 1$	B1	1.1b
	(1)	

Notes

B1 Correct expression for $E(X)$

Mark scheme (b)

Scheme	Marks	AO
$E(X^2) = 25 \times 0.3 + 1 \times 0.25 [+0 \times 0.1] + 25 \times 0.15 + 0.2b^2 [= 11.5 + 0.2b^2]$	M1	1.1b
$"11.5 + 0.2b^2" - ("0.2b - 1")^2 [= 34.26]$	M1	3.1a
$0.16b^2 + 0.4b - 23.76 [= 0] \text{ or } \frac{4}{25}b^2 + \frac{2}{5}b - \frac{594}{25} [= 0]$	M1	1.1b
$b = \underline{11} \text{ [since } b > 5]$	A1	2.2a
	(4)	

Notes

1st M1 Correct attempt at $E(X^2)$ using $\sum x^2 P(X = x)$ at least 3 correct non-zero products

Allow $(-5)^2$ etc

2nd M1 Realising that $\text{Var}(X) = E(X^2) - [E(X)]^2$ needs to be **used**

3rd M1 Reducing their equation to a 3 term quadratic. At least 2 terms correct.

Allow e.g. $0.16b^2 + 0.4b = 23.76$ Condone missing “=0”

A1 For 11 only (from the correct equation) so -13.5 must be eliminated

Correct answer with no incorrect working seen scores 4/4

Mark scheme (c)

Scheme							Marks	AO																																										
<table><tr><td>X</td><td>-5</td><td>-1</td><td>0</td><td>5</td><td>“11”</td></tr><tr><td>X^2</td><td>25</td><td>1</td><td>0</td><td>25</td><td>“121”</td></tr><tr><td>$2 - 3X$</td><td>17</td><td>5</td><td>2</td><td>-13</td><td>“-31”</td></tr><tr><td>$X^2 - 2$</td><td>23</td><td>-1</td><td>-2</td><td>23</td><td>“119”</td></tr><tr><td>$-3X$</td><td>15</td><td>3</td><td>0</td><td>15</td><td>“-33”</td></tr><tr><td>$X^2 + 3X$</td><td>10</td><td>-2</td><td>0</td><td>40</td><td>“154”</td></tr><tr><td>$X^2 + 3X - 2$</td><td>8</td><td>-4</td><td>-2</td><td>38</td><td>“152”</td></tr></table>							X	-5	-1	0	5	“11”	X^2	25	1	0	25	“121”	$2 - 3X$	17	5	2	-13	“-31”	$X^2 - 2$	23	-1	-2	23	“119”	$-3X$	15	3	0	15	“-33”	$X^2 + 3X$	10	-2	0	40	“154”	$X^2 + 3X - 2$	8	-4	-2	38	“152”	M1 A1ft	2.1 1.1b
X	-5	-1	0	5	“11”																																													
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$-3X$	15	3	0	15	“-33”																																													
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$= \underline{0.35}$							A1	1.1b																																										
							(4)																																											
							Total 9																																											

Notes

1st M1 At least 4 values correct for $(X^2 \text{ and } 2 - 3X)$ or for $(X^2 - 2 \text{ and } -3X)$ or $X^2 + 3X$ or $X^2 + 3X - 2$ (o.e.) Allow for solving equation with one sign error

1st A1ft All correct or correct ft with their b but must have $b > 5$ (accurate to 1 sf)

Allow solving equation to get awrt -3.6 **and** awrt 0.56 or $\frac{-3 \pm \sqrt{17}}{2}$ (ft their $b > 5$)

If there are omissions **but no errors** in the lists of values then if 2nd M1 and 2nd A1 are scored then the 1st M1 and 1st A1 can be given by implication.

2nd M1 For identifying the correct values of X required i.e. $X = -1$ and $X = 0$

2nd A1 0.35

NB It is possible to score M0A0M1A1 here if their table of values is incorrect

Correct answer with no incorrect working seen scores 4/4

(Allow correct use of their $b > 5$)