

QQQ – Further Pure 2 – Chapter 6 – Integration Techniques (Version 1)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1. A curve has equation

$$y = \cosh x, \quad 1 \leq x \leq \ln 5$$

Find the exact length of this curve. Give your answer in terms of e.

(5)

(Total 5 marks)

Edexcel FP3 June 2015 Q2

2.

$$I_n = \int_0^2 (x-2)^n e^{4x} dx \quad n \geq 0$$

(a) Prove that for $n \geq 1$

$$I_n = -a^{n-2} - \frac{n}{4} I_{n-1}$$

where a is a constant to be determined.

(4)

(b) Hence determine the exact value of

$$\int_0^2 (x-2)^2 e^{4x} dx$$

(3)

(Total 7 marks)

Edexcel A2 June 2023 Q8

3.

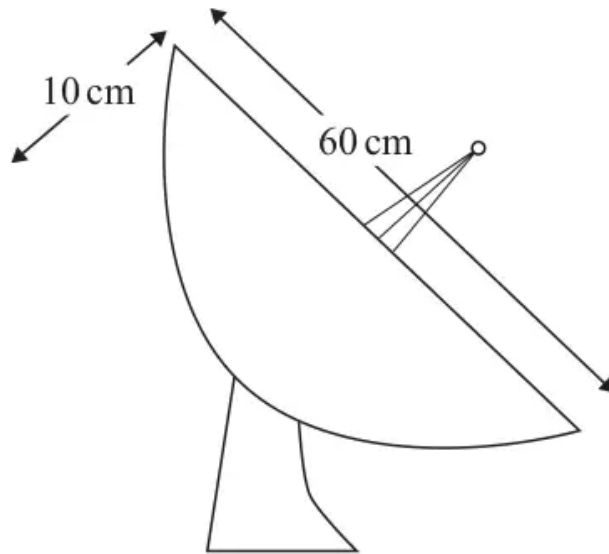


Figure 1

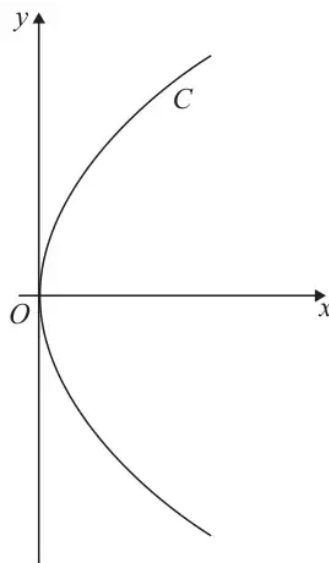


Figure 2

Figure 1 shows a satellite dish.

Figure 2 shows a sketch of the curve C with equation

$$y^2 = Ax \quad 0 \leq x \leq 10$$

where A is a positive constant.

The curved inner surface of the satellite dish is modelled by the surface of revolution formed by rotating curve C through π radians about the x -axis.

The inner surface of the satellite dish has

- a largest diameter of 60 cm

- a depth of 10 cm

as shown in Figure 1.

- (a) Determine the value of A . (2)
- (b) Using algebraic integration, determine, in cm^2 , the area of the curved inner surface of the satellite dish, according to the model. Give your answer to 2 significant figures. (7)
- (c) State a limitation of the model. (1)

(Total 10 marks)

Solutions

1.

Scheme	Marks
$y = \cosh x \Rightarrow \frac{dy}{dx} = \sinh x$	B1
$\int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int \sqrt{1 + \sinh^2 x} dx$	M1
$= \int \cosh x dx$ or $\int \frac{e^x + e^{-x}}{2} dx$	A1
$= [\sinh x]_1^{\ln 5} = \sinh(\ln 5) - \sinh(1)$	dM1
$= \frac{12}{5} - \frac{1}{2} \left(e - \frac{1}{e}\right)$	Alcso
	(5)
	(5 marks)

Notes

B1: Correct derivative

M1: Uses the correct formula with their $\frac{dy}{dx}$

A1: Correct integral (Condone omission of dx)

dM1: $\int \cosh x dx = \sinh x$ and correct use of the correct limits. **Dependent on the first method mark.**

Alcso: Or equivalent (must be in terms of e with no $\ln$'s) **Score when a correct answer is first seen and isw.**

Alternative for first 2 marks

$$y = \frac{e^x + e^{-x}}{2} \Rightarrow \frac{dy}{dx} = \frac{e^x - e^{-x}}{2} = \text{B1}$$

$$\int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int \sqrt{1 + \left(\frac{e^x - e^{-x}}{2}\right)^2} dx = \text{M1}$$

Then apply the scheme

Some equivalent final answers

$$\frac{12}{5} - \frac{e}{2} + \frac{e^{-1}}{2}, \quad 2.4 - \frac{e - e^{-1}}{2}, \quad \frac{12}{5} - \frac{e^2 - 1}{2e}, \quad \frac{24e - 5e^2 + 5}{10e}$$

Special Case: $\frac{dy}{dx} = -\sinh x$ leads to a correct answer. This scores a maximum of 3/5 i.e.

B0M1A1(recovery)dM1A0

2.

Mark scheme (a)

Scheme	Marks	AO
$I_n = \int_0^2 (x-2)^n e^{4x} dx = \left[(x-2)^n \times Ae^{4x} \right]_0^2 - \int_0^2 n(x-2)^{n-1} \times Ae^{4x} dx$ $\left\{ = \left[(x-2)^n \times \frac{1}{4}e^{4x} \right]_0^2 - \int_0^2 n(x-2)^{n-1} \times \frac{1}{4}e^{4x} dx \right\}$	M1	1.1b
$= (0 - A(-2)^n) - A \int_0^2 (x-2)^{n-1} e^{4x} dx$ $\left\{ = \left(0 - \frac{1}{4}(-2)^n \right) - \frac{n}{4} \int_0^2 (x-2)^{n-1} e^{4x} dx \right\}$	M1	1.1b
$= -\frac{1}{(-2)^2}(-2)^n - \frac{n}{4}I_{n-1}$	M1	3.1a
$I_n = -(-2)^{n-2} - \frac{n}{4}I_{n-1}$	A1	2.1
	(4)	

Notes

M1: Applies integration by parts to achieve the correct form

M1: Substitutes in the limits 0 and 2 and simplifies integral to match the form of I_n

M1: Writes 4 as $(-2)^2$ in the first terms (or correctly combines powers) and replaces integral by I_{n-1}

In the alternative award this M for a full process to rearrange to the form I_{n+1} as well as the above constraints.

A1: Achieves the correct answer following correct working, cso. In the alternative a replacement of n by $n - 1$ must also occur. isw

Alternative (a)

Scheme	Marks	AO
$I_n = \int_0^2 (x-2)^n e^{4x} dx = \left[\frac{(x-2)^{n+1}}{n+1} \times e^{4x} \right]_0^2 - \int_0^2 \frac{(x-2)^{n+1}}{n+1} \times Ae^{4x} dx$ $\left\{ = \left[\frac{(x-2)^{n+1}}{n+1} \times e^{4x} \right]_0^2 - \int_0^2 \frac{(x-2)^{n+1}}{n+1} \times 4e^{4x} dx \right\}$	M1	1.1b

$= \left(0 - \frac{(-2)^{n+1}}{n+1}\right) - \frac{A}{n+1} \int_0^2 (x-2)^{n+1} e^{4x} dx$ $\left\{ = \left(0 - \frac{(-2)^{n+1}}{n+1}\right) - \frac{4}{n+1} \int_0^2 (x-2)^{n+1} e^{4x} dx \right\}$	M1	1.1b
$\Rightarrow (n+1)I_n = -(-2)^{n+1} - 4I_{n+1} \Rightarrow I_{n+1} = -\frac{1}{(-2)^2}(-2)^{n+1} - \frac{n+1}{4}I_n$	M1	3.1a
$\Rightarrow I_n = -(-2)^{n-2} - \frac{n}{4}I_{n-1}$	A1	2.1
	(4)	

Mark scheme (b)

Scheme	Marks	AO
$I_2 = \int_0^2 (x-2)^2 e^{4x} dx = -(-2)^0 - \frac{2}{4}I_1 = -1 - \frac{1}{2} \left(-(-2)^{-1} - \frac{1}{4}I_0 \right)$	M1	2.1
$= -1 - \frac{1}{4} + \frac{1}{8} \left(\frac{1}{4}e^8 - \frac{1}{4} \right)$	M1	1.1b
$= \frac{1}{32}e^8 - \frac{41}{32}$	A1	1.1b
	(3)	
	(7 marks)	

Notes

M1: For a full process of reducing the integral to an expression in I_0

M1: Evaluates I_0 and substitutes into the expression.

A1: cso $\frac{1}{32}e^8 - \frac{41}{32}$ or simplified equivalent.

Note: Candidates might use

M1: $I_2 = \int_0^2 (x-2)^2 e^{4x} dx = -(-2)^0 - \frac{2}{4}I_1 = -1 - \frac{1}{2}I_1$

M1: $I_1 = \int_0^2 (x-2)e^{4x} dx = -\frac{1}{16}e^8 + \frac{9}{16}$ therefore $I_2 = -1 - \frac{1}{2} \left(-\frac{1}{16}e^8 + \frac{9}{16} \right)$

A1: cso $\frac{1}{32}e^8 - \frac{41}{32}$ or simplified equivalent. isw

3.

Mark scheme (a)

Scheme	Marks	AO
$30^2 = A(10) \Rightarrow A = \dots$	M1	3.3
$A = 90$	A1	1.1b
	(2)	

Notes

M1: Uses the model $x = 10$ and $y = 30$ to find a value for A

A1: Correct value for A

Mark scheme (b)

Scheme	Marks	AO
$2y \frac{dy}{dx} = \text{their } 90 \text{ or } \frac{dy}{dx} = \frac{\sqrt{90}}{2} x^{-\frac{1}{2}} \text{ oe}$	B1ft	3.4
$SA = 2\pi \int y \sqrt{1 + \left(\frac{45}{y}\right)^2} dx$	M1	3.4
$SA = 2\pi \int y \sqrt{\frac{y^2 + 2025}{y^2}} dx = 2\pi \int \sqrt{y^2 + 2025} dx =$ $2\pi \int \sqrt{90x + 2025} dx$	M1	3.1a
$\int (\alpha x + \beta)^{0.5} dx = K(\alpha x + \beta)^{1.5}$ (NB α may be 1 if they take the 90 out)	M1	1.1b
$\int (90x + 2025)^{0.5} dx = \frac{(90x+2025)^{1.5}}{90 \times 1.5}$ oe e.g. $\frac{3\sqrt{10}(x+22.5)^{1.5}}{1.5}$ FT is $\frac{\sqrt{A}(4x+A)^{1.5}}{12} = \frac{2\sqrt{A}(x+\frac{A}{4})^{1.5}}{3}$ with their A	A1ft	1.1b
Use of correct limits $x = 0$ and $x = 10$ $\{2\pi\} \left[\frac{(90(10)+2025)^{1.5}}{135} - \frac{(90(0)+2025)^{1.5}}{135} \right]$	M1	3.4
$SA = \text{AWRT } 3100 \text{ (cm}^2\text{)}$	A1	1.1b
	(7)	

Notes

B1: Correct derivative.

M1: Uses the model, their $\frac{dy}{dx}$ (which must not be a constant) and the surface area of revolution formula

$SA = 2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ to form an expression of the inner surface area. The 2π must be present either here or later, but may be missing in between. If never considers this mark and the final A will be lost but the in between marks may be gained. Must be integral with respect to x . Attempting to integrate wrt y will likely gain no further marks in the question as they will not obtain correct forms.

M1: Uses correct algebra to manipulate their integral into the form $\int (\alpha x + \beta)^{0.5} dx$ ($\alpha, \beta \neq 0$)

M1: Integrates to the correct form $\int (\alpha x + \beta)^{0.5} dx = K(\alpha x + \beta)^{1.5}$ (allow $\beta = 0$ for this mark)

A1ft: Correct integration, follow through on their value of A – not on incorrect manipulation.

M1: Use of correct limits $x = 0$ and $x = 10$ on an attempt at an integral, need not be correct but must have

come from an attempt at $\int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

A1: Correct answer **awrt** 3100 (units not required).

Mark scheme (c)

Scheme	Marks	AO
Score for any reasonable comment about a limitation of the model. Eg the dish will not be smooth, the curve may not be accurate, the measurements might not have been exact.	B1	3.5b
	(1)	
	(10 marks)	

Notes

B1: Any appropriate comment referencing the model. E.g. their may be imperfections in the dish, it may not be smooth. The answer must be referring to the model of the curve of the dish for the inner surface, not other features of the satellite.

Do **not** accept answer such as “there is an antenna” which do not in any way refer to the curve unless the specifically refer to how it might affect the inner surface.

Do **not** accept answer about the thickness of the dish – the model is specifically referring to the curved inner surface of the dish, so thickness is not relevant to this aspect of the model.