

QQQ – Further Pure 2 – Chapter 5 – Matrix Algebra (Version 1)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 2 & 2 \end{pmatrix}$$

(a) find the characteristic equation for the matrix $\mathbf{A}$, simplifying your answer. (2)

(b) Hence find an expression for the matrix $\mathbf{A}^{-1}$ in the form $\lambda\mathbf{A} + \mu\mathbf{I}$, where λ and μ are constants to be found. (3)

(Total 5 marks)

Edexcel AS June 2019 Q1

2.

$$\mathbf{M} = \begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}$$

(a) Show that 7 is an eigenvalue of the matrix $\mathbf{M}$ and find the other two eigenvalues of $\mathbf{M}$. (5)

(b) Find an eigenvector corresponding to the eigenvalue 7. (4)

(Total 9 marks)

Edexcel FP3 June 2009 Q3

3.

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$$

(a) Determine the eigenvalues of matrix $\mathbf{A}$. (3)

(b) Hence determine an orthogonal matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that

$$\mathbf{D} = \mathbf{P}^T \mathbf{A} \mathbf{P}$$

(5)

(Total 8 marks)

Edexcel A2 June 2025 Q2

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$\det \begin{pmatrix} 3 - \lambda & 2 \\ 2 & 2 - \lambda \end{pmatrix} = (3 - \lambda)(2 - \lambda) - 4 (= 0)$	M1	1.1b
$\lambda^2 - 5\lambda + 2 = 0$	A1	1.1b
	(2)	

Notes

M1: Complete method to find the characteristic equation, condone missing = 0

A1: Obtains a correct three term quadratic equation – may use any variable.

Mark scheme (b)

Scheme	Marks	AO
$\mathbf{A}^2 - 5\mathbf{A} + 2\mathbf{I} = 0$	Blft	1.1b
Multiplies through by $\mathbf{A}^{-1}$ $\mathbf{A} - 5\mathbf{I} + 2\mathbf{A}^{-1} = 0$ and rearranges to get $\mathbf{A}^{-1} = \dots$ OR Rearranges to make $\mathbf{I}$ the subject, takes out a factor of $\mathbf{A}$ and rearranges to get $\mathbf{A}^{-1} = \dots$ $\mathbf{I} = \frac{(5\mathbf{A} - \mathbf{A}^2)}{2} = \mathbf{A} \frac{(5\mathbf{I} - \mathbf{A})}{2} \Rightarrow \mathbf{A}^{-1} = \dots$ OR Rearranges to make $\mathbf{I}$ the subject and multiplies through by $\mathbf{A}^{-1}$ $\mathbf{I} = \frac{5}{2}\mathbf{A} - \frac{1}{2}\mathbf{A}^2 \Rightarrow \mathbf{A}^{-1} = \frac{5}{2}\mathbf{A}\mathbf{A}^{-1} - \frac{1}{2}\mathbf{A}^2\mathbf{A}^{-1}$	M1	3.1a
Identifies $\mathbf{A}^{-1} = -\frac{1}{2}\mathbf{A} + \frac{5}{2}\mathbf{I}$	A1	1.1b
	(3)	
	(5 marks)	

Notes

B1ft: Uses Cayley Hamilton Theorem to produce equation replacing λ with $\mathbf{A}$ and constant term with constant multiple of the identity matrix $\mathbf{I}$

M1: A complete method using part (a) to find $\mathbf{A}^{-1}$

Multiplies through by $\mathbf{A}^{-1}$ and rearranges to get $\mathbf{A}^{-1} = \dots$

Or rearranges to make $\mathbf{I}$ the subject, takes out a factor of $\mathbf{A}$, and rearranges to get $\mathbf{A}^{-1} = \dots$

Or rearranges to make $\mathbf{I}$ the subject and multiplies through by $\mathbf{A}^{-1}$ to get $\mathbf{A}^{-1} = \dots$

A1: Correct expression for $\mathbf{A}^{-1}$, must be using their answer to part (a).

2.

Mark scheme (a)

Scheme	Marks
$\begin{vmatrix} 6-\lambda & 1 & -1 \\ 0 & 7-\lambda & 0 \\ 3 & -1 & 2-\lambda \end{vmatrix} = 0 \quad \therefore (6-\lambda)(7-\lambda)(2-\lambda) + 3(7-\lambda) = 0$	M1
$(7-\lambda) = 0$ verifies $\lambda = 7$ is an eigenvalue (can be seen anywhere)	M1
$\therefore (7-\lambda) \{12 - 8\lambda + \lambda^2 + 3\} = 0 \quad \therefore (7-\lambda) \{\lambda^2 - 8\lambda + 15\} = 0$	A1
$\therefore (7-\lambda)(\lambda-5)(\lambda-3) = 0$ and 3 and 5 are the other two eigenvalues	M1 A1
	(5)

Mark scheme (b)

Scheme	Marks
Set $\begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 7 \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 3 & -1 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	M1
Solve $-x + y - z = 0$ and $3x - y - 5z = 0$ to obtain $x = 3z$ or $y = 4z$ and a second equation which can contain 3 variables	M1 A1
Obtain eigenvector as $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ (or multiple)	A1
	(4)
	(9 marks)

3.

Mark scheme (a)

Scheme	Marks	AO
$\begin{vmatrix} 1-\lambda & -2 \\ -2 & 4-\lambda \end{vmatrix} = 0$ leading to $(1-\lambda)(4-\lambda) - 4 = 0$	M1	1.1b
Solves $\lambda^2 - 5\lambda = 0 \Rightarrow \lambda = \dots$	dM1	1.1b
$\lambda = 5, 0$	A1	1.1b
	(3)	

Notes

M1: Attempts $\begin{vmatrix} 1-\lambda & -2 \\ -2 & 4-\lambda \end{vmatrix} = 0$

dM1: Dependent on the previous method mark. Solves their 3TQ to find a value for λ

A1: Correct values for λ

Mark scheme (b)

Scheme	Marks	AO
$\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \text{their '0'} \begin{pmatrix} x \\ y \end{pmatrix}$ leading to $x - 2y = 0 \Rightarrow x = 2y$ or $\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \text{their '5'} \begin{pmatrix} x \\ y \end{pmatrix}$ leading to $x - 2y = 5x \Rightarrow 2y = -4x$ or $-2x + 4y = 5y \Rightarrow y = -2x$	M1	3.1a
Eigenvectors $\lambda = 0 : \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda = 5 : \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ oe	A1 A1	2.2a 2.2a
$\text{eg } \mathbf{P} = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ 1 & -2 \end{pmatrix}$ or $\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 5 \end{pmatrix}$	B1ft	2.2a
$\text{eg } \mathbf{P} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -2 & 1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 5 & 0 \\ 0 & 0 \end{pmatrix}$	A1ft	2.5

	(5)	
	(8 marks)	

Notes

Note this appears on epen as M1A1A1A1A1 but is being marked as M1A1A1B1A1

M1: Uses $\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \text{'their } \lambda \text{' } \begin{pmatrix} x \\ y \end{pmatrix}$ to form an equation of the form $ax = by$ for at least one of their eigenvalues. May be implied by a correct eigenvector.

A1: Deduces one correct eigenvector for one of the (correct) eigenvalues. Accept any non-zero multiple.

A1: Deduces both correct eigenvectors for the correct eigenvalues. Accept any non-zero multiples.

B1ft: Note: not dependent on the method mark. Deduces a correct matrix **P** or **D**, following through on their eigenvalues or **non-zero** eigenvectors. This may be scored for a correct **D** even if no attempt at the eigenvectors has been made, or a correct f.t. **P** even if the method for eigenvectors was incorrect.

A1ft: Depends on the M having been scored. Correct matrices **D** and **P** which are consistent. Follow through on their eigenvalues and **non-zero** eigenvectors.

Note if they assume the eigenvector for $\lambda = 0$ is **0** then do not allow this for the follow through mark(s) (though the first may be gained for correct **D**).

SC: If they mislabel **P** and **D** then allow A1ftA0ft for both correct but the wrong order, but A0A0 if only one is “correct” but wrong order.