

QQQ – Further Pure 2 – Chapter 4 – Recurrence Relations (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1.

(a) Determine the general solution of the recurrence relation

$$u_n = 2u_{n-1} - u_{n-2} + 2^n \quad n \geq 2$$

(4)

(b) Hence solve this recurrence relation given that $u_0 = 2u_1$ and $u_4 = 3u_2$

(2)

(Total 6 marks)

2.

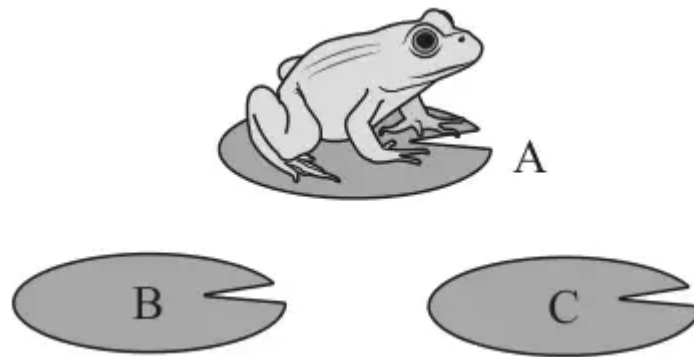


Figure 1

There are three lily pads on a pond. A frog hops repeatedly from one lily pad to another.

The frog starts on lily pad A, as shown in Figure 1.

In a model, the frog hops from its position on one lily pad to either of the other two lily pads with equal probability.

Let p_n be the probability that the frog is on lily pad A after n hops.

(a) Explain, with reference to the model, why $p_1 = 0$ (1)

The probability p_n satisfies the recurrence relation

$$p_{n+1} = \frac{1}{2}(1 - p_n) \quad n \geq 1 \quad \text{where } p_1 = 0$$

(b) Prove by induction that, for $n \geq 1$

$$p_n = \frac{2}{3} \left(-\frac{1}{2} \right)^n + \frac{1}{3}$$

(6)

(c) Use the result in part (b) to explain why, in the long term, the probability that the frog is on lily pad A is $\frac{1}{3}$ (1)

(Total 8 marks)

3. A student takes out a loan for £1000 from a bank.

The bank charges 0.5% monthly interest on the amount of the loan yet to be repaid.

At the end of each month

- the interest is added to the loan
- the student then repays £50

Let U_n be the amount of money owed n months after the loan was taken out.

The amount of money owed by the student is modelled by the recurrence relation

$$U_n = 1.005U_{n-1} - A \quad U_0 = 1000 \quad n \in \mathbb{Z}^+$$

where A is a constant.

- (a)** **(i)** State the value of the constant A .
- (ii)** Explain, in the context of the problem, the value 1.005

(2)

Using the value of A found in part (a)(i),

- (b)** solve the recurrence relation

$$U_n = 1.005U_{n-1} - A \quad U_0 = 1000 \quad n \in \mathbb{Z}^+$$

(5)

- (c)** Hence determine, according to the model, the number of months it will take to completely repay the loan.

(2)

(Total 9 marks)