

QQQ – Further Pure 2 – Chapter 4 – Recurrence Relations (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. The general solution of the first order recurrence relation

$$u_{n+1} + au_n = bn^2 + cn + d \quad n \geq 0$$

is given by

$$u_n = A(3)^n + 5n^2 + 1$$

where A is an arbitrary non-zero constant.

By considering expressions for u_{n+1} and u_n , find the values of the constants a, b, c and d . **(3)**

(Total 3 marks)

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2. Determine a closed form for the recurrence system

$$u_1 = 4 \quad u_2 = 6$$

$$u_{n+2} = 6u_{n+1} - 9u_n \quad (n = 1, 2, 3, \dots)$$

(Total 5 marks)

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3. A recurrence system is defined by

$$u_{n+2} = 9(n+1)^2 u_n - 3u_{n+1} \quad n \geq 1$$

$$u_1 = -3, \quad u_2 = 18$$

Prove by induction that, for $n \in \mathbb{N}$,

$$u_n = (-3)^n n!$$

(6)

(Total 6 marks)

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4. A person takes a course of a particular vitamin.

Before the course there was none of the vitamin in the person's body.

During the course, vitamin tablets are taken at the same time each day.

Initially two tablets are taken and on each following day only one tablet is taken.

Each tablet contains 10 mg of the vitamin.

Between doses the amount of the vitamin in the person's body decreases naturally by 60%

Let u_n mg be the amount of the vitamin in the person's body immediately after a tablet is taken, n days after the initial two tablets were taken.

(a) Explain why u_n satisfies the recurrence relation

$$u_0 = 20 \quad u_{n+1} = 0.4u_n + 10$$

(2)

The general solution to this recurrence relation has the form $u_n = a(0.4)^n + b$

(b) Determine the value of a and the value of b .

(4)

The course is only effective if the amount of the vitamin in the person's body remains above 6 mg at **all times** throughout the course.

(c) Determine whether this course of the vitamin will be effective for this person, giving a reason for your answer.

(3)

(Total 9 marks)