

QQQ – Further Pure 2 – Chapter 2 – Groups (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. The set $G = \{1, 3, 7, 9, 11, 13, 17, 19\}$ under the binary operation of multiplication modulo 20 forms a group.

- (a) Find the inverse of each element of G . (3)
- (b) Find the order of each element of G . (3)
- (c) Find a subgroup of G of order 4 (1)
- (d) Explain how the subgroup you found in part (c) satisfies Lagrange's theorem. (1)

(Total 8 marks)

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2. A binary operation $\star$ on the set of non-negative integers, $\mathbb{Z}_0^+$, is defined by

$$m \star n = |m - n| \quad m, n \in \mathbb{Z}_0^+$$

- (a) Explain why $\mathbb{Z}_0^+$ is closed under the operation $\star$ (1)
- (b) Show that 0 is an identity for $(\mathbb{Z}_0^+, \star)$ (2)
- (c) Show that all elements of $\mathbb{Z}_0^+$ have an inverse under $\star$ (2)
- (d) Determine if $\mathbb{Z}_0^+$ forms a group under $\star$, giving clear justification for your answer. (3)

(Total 8 marks)

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3. The set $S = \{1, 3, 5, 9, 11, 13\}$ forms the group G , under the operation multiplication modulo 14

(a) Complete the Cayley table below for the group G

$\times_{14}$	1	3	5	9	11	13
1	1	3	5	9	11	13
3	3	9	1	13	5	11
5	5	1	11			
9	9	13		11		
11	11	5			9	
13	13	11				1

(3)

(b) Write down a subgroup of G of order 2

(1)

The group H is defined by the Cayley table below.

$*$	p	q	r	s	t	u
p	p	q	r	s	t	u
q	q	t	u	r	s	p
r	r	u	t	q	p	s
s	s	r	q	p	u	t
t	t	s	p	u	r	q
u	u	p	s	t	q	r

(c) Show that G and H are isomorphic.

(3)

(Total 7 marks)