

QQQ – Further Pure 2 – Chapter 1 – Number Theory (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1.

(a) Use the Euclidean algorithm to show that 124 and 17 are relatively prime (coprime). **(2)**

(b) Hence solve the equation

$$124x + 17y = 10$$

(3)

(c) Solve the congruence equation

$$124x \equiv 6 \pmod{17}$$

(2)

(Total 7 marks)

Edexcel A2 June 2022 Q4

2.

(i) Making your reasoning clear and using modulo arithmetic, show that

$$214^6 \text{ is divisible by } 8$$

(3)

(ii) The following 7-digit number has four unknown digits

$$\boxed{a} 5 \boxed{b} 8 \boxed{a} \boxed{b} 0$$

Given that the number is divisible by 11

(a) determine the value of the digit a . **(2)**

Given that the number is also divisible by 3

(b) determine the possible values of the digit b . **(3)**

(Total 8 marks)

Edexcel AS June 2023 Q5

3.

(i) A password is to be created from 5 characters using the following criteria

- one capital letter
- 3 distinct single digits
- one of the four symbols @ £ \$ &

Determine how many different passwords can be created if

(a) the characters are in the order of the bullet points above, **(2)**

(b) the characters can be in any order. **(1)**

(ii) Use Fermat's Little Theorem to determine the least positive residue of 4^{50} modulo 13 **(3)**

(iii) Determine the total number of different, positive **odd** numbers, less than 1000, that contain the digit 2 **(4)**

(Total 10 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$124 = 17 \times 7 + 5$ $17 = 3 \times 5 + 2$ $5 = 2 \times 2 + 1$ $\{2 = 2 \times 1\}$	M1	2.1
Since the gcd is 1 , 124 and 17 are relatively prime (coprime)	A1	2.4
	(2)	

Notes

M1: Complete attempt at the Euclidean algorithm to find the gcd (hcf) of 124 and 17, condone a numerical slip.

A1: A fully correct application of the Euclidean algorithm and draws the conclusion that since the **gcd/hcf = 1** therefore 124 and 17 are **relatively prime (coprime)**.

Mark scheme (b)

Scheme	Marks	AO
$1 = 5 - 2 \times 2$ $1 = 5 - 2(17 - 3 \times 5) \Rightarrow 1 = 7 \times 5 - 2 \times 17$ $1 = 7(124 - 17 \times 7) - 2 \times 17$ $\Rightarrow 1 = 7 \times 124 - 51 \times 17$	M1 A1	2.1 1.1b
$\Rightarrow 10 = 70 \times 124 - 510 \times 17$ $x = 70, y = -510$	A1	2.2a
	(3)	

Notes

M1: Attempts to find the Bezout's identity, condone sign slips

A1: Correct Bezout's identity.

A1: Deduces a set of correct values of x and y .

Alternatively

M1: Uses the solution to (a) to find an identity of the form $a \times 124 + b \times 17 = c$

A1: Correct identity

A1: Deduces a set of correct values of x and y .

Alternative

Scheme	Marks	AO
For example $124 = 17 \times 7 + 5$ $5 = 124 - 7 \times 17$	M1 A1	2.1 1.1b
$\Rightarrow 10 = 2 \times 124 - 14 \times 17$ $x = 2, y = -14$	A1	2.2a

Alternative

Scheme	Marks	AO
$1 = 5 - 2 \times 2$ $1 = 5 - 2(17 - 3 \times 5) \Rightarrow 4 = 2 \times 17 - 6 \times 5$ $4 = 2 \times 17 - 6(124 - 17 \times 7)$ $\Rightarrow 4 = 44 \times 17 - 6 \times 124$	M1 A1	2.1 1.1b
$\Rightarrow 10 = 110 \times 17 - 15 \times 124$ $x = -15, y = 110$	A1	2.2a

Mark scheme (c)

Scheme	Marks	AO
A complete method using modulo arithmetic to achieve $x \equiv \dots \pmod{17}$ For example $7 \times 124x \equiv 7 \times 6 \pmod{17} \Rightarrow x \equiv \dots \pmod{17}$	M1	2.1
$x \equiv 8 \pmod{17}$ o.e.	A1	1.1b
	(2)	
	(7 marks)	

Notes

M1: A complete method to reach using modulo arithmetic to achieve $x \equiv \dots \pmod{17}$

For example

Multiplies through by their multiplicative inverse of 124 and reaches $x \equiv \dots \pmod{17}$. Bezout's identity used in part (b) follow through on their multiple of 124

A1: $x \equiv 8 \pmod{17}$ o.e.

2.

Mark scheme (i)

Scheme	Marks	AO
$214 \equiv 6 \pmod{8}$	B1	1.1b
$(214)^2 \equiv 6^2 \pmod{8} \Rightarrow (214)^2 \equiv 4 \pmod{8}$ $(214)^6 \equiv 4^3 \pmod{8} \Rightarrow (214)^6 \equiv 64 \pmod{8}$ or $(214)^6 \equiv 6^6 \pmod{8} \equiv (6^3)^2 \pmod{8} \equiv (0)^2 \pmod{8}$ Or $(214)^3 \equiv 6^3 \pmod{8} \Rightarrow (214)^3 \equiv 0 \pmod{8}$ Or $214 \equiv -2 \pmod{8} \Rightarrow (214)^6 \equiv (-2)^6 \pmod{8} \equiv 64 \pmod{8}$	M1	2.1
$(214)^6 \equiv 0 \pmod{8}$ therefore 214^6 is divisible by 8	A1	2.4
	(3)	

Notes

B1: States $214 \equiv 6 \pmod{8}$

M1: Uses modulo arithmetic to find $(214)^6 \equiv a \pmod{8}$

A1: Achieves $(214)^6 \equiv 0 \pmod{8}$ from correct modulo arithmetic and draws the conclusion that 214^6 is divisible by 8

Mark scheme (ii)(a)

Scheme	Marks	AO
Uses $a - 5 + b - 8 + a - b + 0 = 2a - 13 = 11n$ to find a possible value of a .	M1	1.1b
$a = 1$	A1	2.2a

	(2)	
--	-----	--

Notes

M1: Uses the division rule for 11 to find a value for a .

A1: $a = 1$

Mark scheme (ii)(b)

Scheme	Marks	AO
"1" + 5 + b + 8 + "1" + b + 0 = $2b + 15 = 3n$ to find a possible value of b .	M1	3.1a
Any two of $b = 0, 3, 6$ or 9	A1	1.1b
All four of $b = 0, 3, 6$ and 9	A1	2.2a
	(3)	
	(8 marks)	

Notes

M1: Uses the division rule for 3 to find a value for b .

A1: Any two correct values for b .

A1: All four correct values for b .

3.

Mark scheme (i)(a)

Scheme	Marks	AO
Any two out of 26 or $10 \times 9 \times 8$ or 4 seen multiplied together	M1	1.1b
$26 \times 10 \times 9 \times 8 \times 4 = 74880$	A1	1.1b
	(2)	

Notes

M1: Any two out of 26 or $10 \times 9 \times 8$ or 4 seen multiplied together

A1: Correct answer

Mark scheme (i)(b)

Scheme	Marks	AO
their '74880' $\times \frac{5!}{3!} = 1497600$	B1ft	1.1b
	(1)	

Notes

B1ft: For the correct answer or follow through their answer to (a) multiplied by 20. Note they may start again and use $26 \times {}^{10}C_3 \times 4 \times 5!$

Mark scheme (ii)

Scheme	Marks	AO
$4^{12} \equiv 1 \pmod{13}$ or $4^{13} \equiv 4 \pmod{13}$	B1	1.2
$(4^{12})^4 \times 4^2 \equiv 1 \times 4^2 \pmod{13}$ or $(4^{13})^3 \times 4^{11} \equiv 4^3 \times 4^{11} \equiv 4^{14} \equiv 4^{13} \times 4^1 \equiv \dots \pmod{13}$	M1	2.1
$4^{50} \equiv 3 \pmod{13}$	A1	2.2a
	(3)	

Notes

B1: Recalls Fermat's Little Theorem correctly. May be implicit in the working rather than stated separately. Accept if stated in terms of a and p .

M1: A complete attempt to write 4^{50} as powers of 4^{12} or 4^{13} and completes the process to find the residue.

A1: Deduces the correct residue.

Mark scheme (iii)

Scheme	Marks	AO
The number of different 1-digit odd numbers that do not contain the digit 2 is 5	B1	2.2a
The number of different 2-digit odd numbers that do not contain the digit 2 is $8 \times 5 \{= 40\}$ The number of different 3-digit odd numbers that do not contain the digit 2 is $8 \times 9 \times 5 \{= 360\}$	M1	1.1b

There are 500 odd numbers therefore $500 - (5 + 40 + 360)$	dM1	2.1
$= 95$	A1	1.1b
	(4)	
	(10 marks)	

Notes

B1: Deduces that there are 5 different 1-digit odd numbers that do not contain the digit 2. Variation: if they consider 000 to 999 allow B1 for correct option of 9 for first digit. (corrected from the printed mark scheme: the printed note says 3 different 1-digit odd numbers, but there are 5 (1, 3, 5, 7, 9), as in the scheme)

M1: Finds the number of different 2-digit and 3-digit odd numbers that do not contain the digit 2. (In the variation they would do $9 \times 9 \times 5$ possibilities without 2, giving 405 directly.)

dM1: For $500 -$ the sum of the number of 1, 2, 3 digits that don't contain the digit 2. Must have been full method to find all the digits without 2 first.

A1: Correct answer

Alternative 1

Scheme	Marks	AO
Identifies that 2 can only appear in the first and/or second position	B1	2.2a
First digit is 2, second digit 10 possible, third digit 5 possible $= 10 \times 5 = \dots \{50\}$ First digit not 2, 9 options (including 0), second digit is 2, third digit 5 possible $= 9 \times 5 = \dots \{45\}$	M1	1.1b
Therefore total number of digits is "50" + "45" = ...	dM1	2.1
$= 95$	A1	1.1b
	(4)	

B1: Deduces that there are only two positions for 2, first and second. (May be implied)

M1: Considers first digit is 2 and the possible digits for second and third position and considers second digit is 2 and the possible digits for first and third position. Allow if they have double counted some digits for this mark.

dM1: Full method considering the possibilities for 2 in various positions and adds the results. They must subtract any repeats for this mark.

A1: Correct answer

Alternative 2

Scheme	Marks	AO
--------	-------	----

1 digit number = 0 ways	B1	2.2a
2 digit number = $1 \times 5 = 5$ ways 3 digit number starting with 2 = $1 \times 10 \times 5 = 50$ ways Other 3 digit number with middle number 2 = $8 \times 1 \times 5 = 40$ ways	M1	1.1b
Total number = $0 + 5 + 50 + 40$	dM1	2.1
= 95	A1	1.1b
	(4)	

B1: Deduces that there are no single digit numbers

M1: Considers the possible 2 and 3 digit numbers. Allow if they have double counted some digits for this mark or omit some cases as long as at least some cases are considered.

dM1: Adds up all the possible digits following consideration of all possible combinations. They must subtract any repeats for this mark. For 3 digit numbers they may have $1 \times 10 \times 5$ starting 2, $9 \times 1 \times 5$ with 2 in middle, but need to note 5 of these are repeated (22x), and so get $5 + 50 + 45 - 5 = \dots$

A1: Correct answer.

Other Alternatives are possible. E.g.

B1: Considers 3 digit numbers from 000 to 999. (May be implicit)

M1: Starting with 2 : 50 possibilities (must be odd), middle 2 but not starting 2 : $9 \times 1 \times 5$ possibilities.

dM1: So total number of possibilities is $50 + 45 = \dots$

A1: Correct answer.