

QQQ – Further Pure 1 – Chapter 8 – Numerical Methods (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1.

- (a)** Use Simpson's rule with 4 intervals to find an estimate for

$$\int_0^2 e^{\sin^2 x} dx$$

Give your answer to 3 significant figures. **(4)**

Given that $\int_0^2 e^{\sin^2 x} dx = 3.855$ to 4 significant figures,

- (b)** comment on the accuracy of your answer to part (a). **(1)**

(Total 5 marks)

2.

(a) Given that

$$y = e^{\operatorname{cosec}^2 x}$$

complete the table below with the value of y corresponding to $x = 2$, giving your answer to 2 decimal places.

x	1.5	1.75	2	2.25	2.5
y	2.73	2.81		5.22	16.31

(1)

(b) Use Simpson's rule, with all the values of y in the completed table, to estimate, to one decimal place, the value of

$$\int_{1.5}^{2.5} e^{\operatorname{cosec}^2 x} \mathrm{d}x$$

(3)

(c) Using your answer to part (b) and making your method clear, estimate the value of

$$\int_{1.5}^{2.5} e^{\cot^2 x} \mathrm{d}x$$

(2)

(Total 6 marks)

3. An area of woodland contains a mixture of blue and yellow flowers.

A study found that the proportion, x , of blue flowers in the woodland area satisfies the differential equation

$$\frac{dx}{dt} = \frac{xt(0.8 - x)}{x^2 + 5t} \quad t > 0$$

where t is the number of years since the start of the study.

Given that exactly 3 years after the start of the study half of the flowers in the woodland area were blue,

- (a) use one application of the approximation formula $\left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1} - y_n}{h}$ to estimate the proportion of blue flowers in the woodland area half a year later. **(5)**
- (b) Deduce from the differential equation the proportion of flowers that will be blue in the long term. **(1)**

(Total 6 marks)

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4. The velocity $v \text{ m s}^{-1}$, of a raindrop, t seconds after it falls from a cloud, is modelled by the differential equation

$$\frac{dv}{dt} = -0.1v^2 + 10 \quad t \geq 0$$

Initially the raindrop is at rest.

- (a) Use two iterations of the approximation formula $\left(\frac{dy}{dx}\right)_n \approx \frac{y_{n+1} - y_n}{h}$ to estimate the velocity of the raindrop 1 second after it falls from the cloud. **(5)**

Given that the initial acceleration of the raindrop is found to be smaller than is suggested by the current model,

- (b) refine the model by changing the value of one constant. **(1)**

(Total 6 marks)

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Solutions

1.

Mark scheme (a)

Scheme	Marks	AO																						
Step length = 0.5	B1	1.1b																						
<table><tr><td>x</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td></tr><tr><td rowspan="3">y</td><td>y_0</td><td>y_1</td><td>y_2</td><td>y_3</td><td>y_4</td></tr><tr><td>e^0</td><td>$e^{\sin^2 \frac{1}{2}}$</td><td>$e^{\sin^2 1}$</td><td>$e^{\sin^2 \frac{3}{2}}$</td><td>$e^{\sin^2 2}$</td></tr><tr><td>1</td><td>1.258...</td><td>2.030...</td><td>2.704...</td><td>2.286...</td></tr></table>	x	0	0.5	1	1.5	2	y	y_0	y_1	y_2	y_3	y_4	e^0	$e^{\sin^2 \frac{1}{2}}$	$e^{\sin^2 1}$	$e^{\sin^2 \frac{3}{2}}$	$e^{\sin^2 2}$	1	1.258...	2.030...	2.704...	2.286...	M1	1.1b
x	0	0.5	1	1.5	2																			
y	y_0	y_1	y_2	y_3	y_4																			
	e^0	$e^{\sin^2 \frac{1}{2}}$	$e^{\sin^2 1}$	$e^{\sin^2 \frac{3}{2}}$	$e^{\sin^2 2}$																			
	1	1.258...	2.030...	2.704...	2.286...																			
$\int_0^2 e^{\sin^2 x} dx \approx \frac{0.5}{3} \{y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\} = \frac{0.5}{3} \times \{23.198 \dots\} (= 3.8664 \dots)$	M1	1.1b																						
= 3.87	Alcao	1.1b																						
	(4)																							

Notes

B1: Correct step length of 0.5 which may be implied e.g. by their 0, 0.5, etc.

M1: Attempts to find y values for **all** their x values – may be in terms of e or numerical values. Must be trying to find at least 3 values (e.g. if step length 1 is used). If substitution is not seen at least 2 of the values other than 1 must be correct to at least 2 s.f. rounded or truncated.

M1: Correct application of Simpson's rule: $\frac{h}{3}(\text{ends} + 2\text{evens} + 4\text{odds})$ (must have an odd number of ordinates). Must be y values **not** x values.

Alcao: 3.87. Must be to 3 s.f.

Note: must see evidence of Simpson's Rule so answer with no working scores no marks.

Mark scheme (b)

Scheme	Marks	AO
eg It is accurate to 2 significant figures	B1	3.2b

	(1)	
	(5 marks)	

Notes

B1: Must have awrt 3.87 for part (a), makes a sensible comment regarding the accuracy. Must relate to the accuracy, not just the value.

Accept

- It is correct to 2 significant figures
- It is correct to 1 decimal place
- It is not correct to 3 significant figures
- It is awrt 99.6% accurate (using 3.87) or 99.7% accurate (using 3.866...)
- Percentage error is awrt 0.3%/0.4% so very accurate
- Very accurate as only out by 0.015 (requires reference to accuracy and evidence)

Do not accept e.g. it is out by 0.015 with no reference to what this means about the accuracy. Do not accept e.g. “very close to” as evidence, some quantification (absolute error, number of s.f. it agrees to) must be given.

2.

Mark scheme (a)

Scheme	Marks	AO
3.35 cao	B1	1.1b
	(1)	

Notes

B1: Correct value 3.35 only, this may be seen in the table or in the script

Mark scheme (b)

Scheme	Marks	AO
Step length = 0.25	B1	1.1b
$\frac{1}{3} \times \text{“0.25”} [2.73 + 16.31 + 2(\text{“3.35”}) + 4(2.81 + 5.22)]$	M1	1.1b
= 4.8	A1	1.1b
	(3)	

Notes

B1: Correct step length, may be seen in the application of Simpson's rule formula. Could be seen as

$$1.75 - 1.5 \text{ or } \frac{2.5 - 1.5}{4}$$

M1: Correct application of Simpson's rule using their step length. If no step length stated and uses

$$\frac{1}{3} [\text{ends} + 2(\text{evens}) + 4(\text{odd})] \text{ this is M0}$$

A1: Awrt 4.8 must be using 3.35 from (a)

Note calculator answer is 4.67 is no marks

Note Answer only with no working is no marks,

Note: If they write $\frac{1}{3} \times h [\text{ends} + 2(\text{evens}) + 4(\text{odd})]$ which leads to awrt 4.8 this gets B1M1A1

If their answer is incorrect this would be B0 (if no h stated) M0A0

Mark scheme (c)

Scheme	Marks	AO
$\int_{1.5}^{2.5} e^{\cot^2 x} dx = \int_{1.5}^{2.5} e^{\operatorname{cosec}^2 x - 1} dx = \frac{1}{e} \int_{1.5}^{2.5} e^{\operatorname{cosec}^2 x} dx =$ $\frac{1}{e} \times "4.8"$	M1	3.1a
= awrt 1.8	A1ft	1.1b
	(2)	
	(6 marks)	

Notes

M1: Uses a correct identity to express $\int e^{\cot^2 x} dx$ in terms of $\int e^{\operatorname{cosec}^2 x} dx$ and then uses correct index

work to allow $\int_{1.5}^{2.5} e^{\cot^2 x} dx$ to be evaluated using part (b) $\frac{1}{e} \times "4.8"$

A1ft: Awrt 1.8 or follow through $\frac{1}{e} \times$ their (b) which must be evaluated and correct for their (b) rounded to 1 d.p.

If uses Simpson's rule again this is M0 A0

3.

Mark scheme (a)

Scheme	Marks	AO
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$t_0 = 3$ and step is half a year, so $h = \frac{1}{2}$ and $x_0 = \frac{1}{2}$	B1	3.3
$\left(\frac{dx}{dt}\right)_0 = \frac{0.5 \times 3 \times (0.8 - 0.5)}{0.5^2 + 5 \times 3} = \dots \left(\frac{9}{305} = 0.0295 \dots\right)$	M1 A1	3.4 1.1b
So when $t = 3.5, x \approx \frac{1}{2} + \frac{1}{2} \times \frac{9}{305}, = \dots$	M1	1.1b
$= \frac{157}{305} \approx 0.51475 \dots$	A1	3.2a
	(5)	

Notes

B1: Uses the given information to set up correct parameters for the model, $t_0 = 3, x_0 = \frac{1}{2}$, and $h = \frac{1}{2}$ seen or implied. Look for e.g. $x = \frac{1}{2}$ and $t = 3$ used in the d.e. and $h = \frac{1}{2}$ used in the approximation formula.

M1: Uses their values for x and t in the given equation to find $\left(\frac{dx}{dt}\right)_0$

Condone one slip when substituting their x and their t into the d.e.

This may be implied by $\left(\frac{dx}{dt}\right)_0 = \text{awrt } 0.03$.

May be seen embedded e.g. $x = \frac{1}{2} + \frac{1}{2} \left(\frac{0.5 \times 3 \times (0.8 - 0.5)}{0.5^2 + 5 \times 3} \right)$

or may be seen in a table e.g.

n	x	t	dy/dx
0	$\frac{1}{2}$	3	0.0295...
1			

A1: For $\left(\frac{dx}{dt}\right)_0 = \frac{9}{305}$ or awrt 0.03. May be implied by subsequent work or seen as $\frac{0.45}{15.25}$

M1: Applies the approximation formula with their x , their h and their $\left(\frac{dx}{dt}\right)_0$ to find a value for x .

A1: Correct proportion found, accept as fraction or awrt 0.515 or e.g. 51.5%

Mark scheme (b)

Scheme	Marks	AO
Long term proportion is $\frac{4}{5}$	B1	3.4

	(1)	
	(6 marks)	

Notes

B1: Deduces the correct long term proportion. Allow equivalents e.g. 0.8 or 80%

4.

Mark scheme (a)

Scheme	Marks	AO
Identifies $t_0 = 0$, $v_0 = 0$, $\left(\frac{dv}{dt}\right)_0 = 10$ and $h = 0.5$	B1	3.4
$v_1 = v_0 + h \left(\frac{dv}{dt}\right)_0 \Rightarrow v_1 = 0 + 0.5 \times 10 = \dots$	M1	1.1b
$v_1 = 5$	A1	1.1b
$\left(\frac{dv}{dt}\right)_1 = -0.1(5)^2 + 10 = \dots \{7.5\}$ $v_2 = v_1 + h \left(\frac{dv}{dt}\right)_1 \Rightarrow v_2 = 5 + 0.5 \times 7.5 = \dots$	M1	3.4
$v_2 = 8.75$ so 8.75 m s^{-1}	A1	1.1b
	(5)	

Notes

B1: Uses the model to identify the correct initial conditions and requirements for h . May be implied by use in the equation.

M1: Applies the approximation formula with their values for v_0 , $\left(\frac{dv}{dt}\right)_0$ and h to find a value for v_1

A1: $v_1 = 5$

M1: Uses their v_1 to find a value for $\left(\frac{dv}{dt}\right)_1$ and applies the approximation formula with their values for

v_1 , $\left(\frac{dv}{dt}\right)_1$ and h to find a value for v_2

A1: $v_2 = 8.75$ or 8.75 m s^{-1}

Mark scheme (b)

Scheme	Marks	A0
$\frac{dv}{dt} = -0.1v^2 + A$ where $0 < A < 10$	B1	3.5c
	(1)	
	(6 marks)	

Notes

B1: Reduce the value of 10 or explains this is what needs **reducing**, but do not accept 0 or negative values in place of the 10. Note: “change the 10” is B0 if it does not explain how to change it.