

QQQ – Further Pure 1 – Chapter 8 – Numerical Methods (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1.

$$\frac{dy}{dx} = ye^{x^2}.$$

It is given that $y = 0.2$ at $x = 0$.

- (a) Use the approximation $\frac{y_1 - y_0}{h} \approx \left(\frac{dy}{dx}\right)_0$, with $h = 0.1$, to obtain an estimate of the value of y at $x = 0.1$. **(2)**

- (b) Use your answer to part (a) and the approximation $\frac{y_2 - y_0}{2h} \approx \left(\frac{dy}{dx}\right)_1$, with $h = 0.1$, to obtain an estimate of the value of y at $x = 0.2$.
Gives your answer to 4 decimal places. **(3)**

(Total 5 marks)

2.

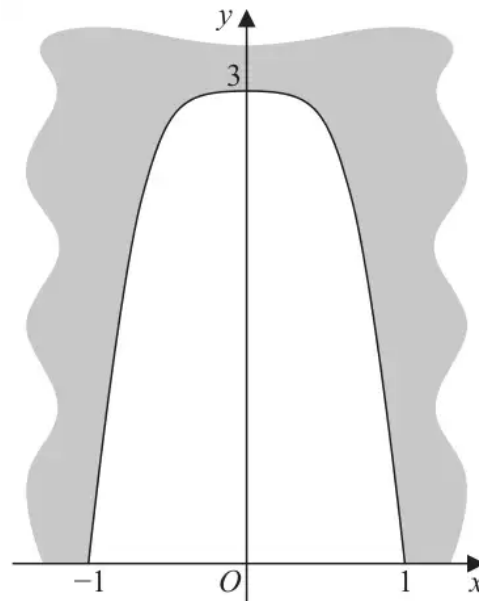


Figure 1

Figure 1 shows a sketch of the vertical cross-section of the entrance to a tunnel. The width at the base of the tunnel entrance is 2 metres and its maximum height is 3 metres.

The shape of the cross-section can be modelled by the curve with equation $y = f(x)$ where

$$f(x) = 3 \cos \left(\frac{\pi}{2} x^2 \right) \quad x \in [-1, 1]$$

A wooden door of uniform thickness 85 mm is to be made to seal the tunnel entrance.

Use Simpson's rule with 6 intervals to estimate the volume of wood required for this door, giving your answer in m^3 to 4 significant figures.

(6)

(Total 6 marks)

3. A population of deer was introduced onto an island.

The number of deer, P , on the island at time t years following their introduction is modelled by the differential equation

$$\frac{dP}{dt} = \frac{P}{5000} \left(1000 - \frac{P(t+1)}{6t+5} \right) \quad t > 0$$

It was estimated that there were 540 deer on the island six months after they were introduced.

Use **two** applications of the approximation formula $\left(\frac{dy}{dx} \right)_n \approx \frac{y_{n+1} - y_n}{h}$ to estimate the number of deer on the island 10 months after they were introduced.

(7)

(Total 7 marks)

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4. The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} = 2y^2 - x - 1$$

where $\frac{dy}{dx} = 3$ and $y = 0$ at $x = 0$

Use the approximations

$$\left(\frac{d^2y}{dx^2} \right)_n \approx \frac{(y_{n+1} - 2y_n + y_{n-1}))}{h^2} \quad \text{and} \quad \left(\frac{dy}{dx} \right)_n \approx \frac{(y_{n+1} - y_{n-1}))}{2h}$$

with $h = 0.1$ to find an estimate for the value of y at $x = 0.2$

(7)

(Total 7 marks)

Edexcel AS October 2020 Q1

Solutions

1.

Mark scheme (a)

Scheme	Marks
$\frac{y_1 - 0.2}{0.1} \approx \left(\frac{dy}{dx}\right)_0 = 0.2 \times e^0 (= 0.2)$	M1
$y_1 \approx 0.22$	A1
	(2)

Mark scheme (b)

Scheme	Marks
$\left(\frac{dy}{dx}\right)_1 \approx 0.22 \times e^{0.01} \approx 0.2222 \dots$	B1
$\frac{y_2 - 0.2}{0.2} \approx 0.2222 \dots$	M1
$y_2 \approx 0.2444$ cao	A1
	(3)
	(5 marks)

2.

Scheme	Marks	AO																								
Step $\frac{1}{3}$	B1	1.1b																								
<table><tr><td></td><td>y_0</td><td>y_1</td><td>y_2</td><td>y_3</td><td>y_4</td><td>y_5</td><td>y_6</td></tr><tr><td>x</td><td>-1</td><td>$-\frac{2}{3}$</td><td>$-\frac{1}{3}$</td><td>0</td><td>$\frac{1}{3}$</td><td>$\frac{2}{3}$</td><td>1</td></tr><tr><td>y</td><td>0</td><td>2.2981</td><td>2.9544</td><td>3</td><td>2.9544</td><td>2.2981</td><td>0</td></tr></table>		y_0	y_1	y_2	y_3	y_4	y_5	y_6	x	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	y	0	2.2981	2.9544	3	2.9544	2.2981	0	M1	3.4
	y_0	y_1	y_2	y_3	y_4	y_5	y_6																			
x	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1																			
y	0	2.2981	2.9544	3	2.9544	2.2981	0																			
$y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6 = \text{“42.203”}$ $\{0 + 4(2.2981 + 3 + 2.2981) + 2(2.9544 + 2.9544) + 0\}$	M1	1.1b																								

$= 42.203 \left(= 24 \cos \left(\frac{2\pi}{9} \right) + 12 \cos \left(\frac{\pi}{18} \right) + 12 \right)$	A1	1.1b
So volume required is approx. $\frac{85}{1000} \times \frac{1}{3} \times "42.203"$	M1	3.1a
$= \text{awrt } 0.3986 \text{ m}^3$	A1	3.2a
	(6)	
	(6 marks)	

Notes

B1: Correct strip width for the method chosen $\frac{1}{3}$ for the interval $[-1, 1]$

M1: Uses the model to find the appropriate values for the method. May use that the function is even to only work out half of them, so may be implied by use in the formula. At least two correct values to 4 s.f. needed for the method.

M1: Applies the "bracket" of Simpson's rule, " $y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6$ ". Coefficients must be correct.

A1: Correct value for the "bracket". If not explicitly seen, may be implied by awrt 4.689 as a value for the cross section area following correct values.

M1: Correct full method to find the volume. E.g. multiplies their bracket by their $\frac{h}{3}$ and by 0.085.

Accept an attempt in any consistent units, so e.g. in mm^3 ie $85 \times \frac{1}{3} \times "42.203" \times 1000^2$

A1: Correct answer in m^3 .

Alternative interval [0,1]

Scheme	Marks	AO																								
step $\frac{1}{6}$ and the answer is doubled later	B1	1.1b																								
<table><tr><td></td><td>y_0</td><td>y_1</td><td>y_2</td><td>y_3</td><td>y_4</td><td>y_5</td><td>y_6</td></tr><tr><td>x</td><td>0</td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{2}$</td><td>$\frac{2}{3}$</td><td>$\frac{5}{6}$</td><td>1</td></tr><tr><td>y</td><td>3</td><td>2.9971</td><td>2.9544</td><td>2.7716</td><td>2.2981</td><td>1.3852</td><td>0</td></tr></table>		y_0	y_1	y_2	y_3	y_4	y_5	y_6	x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1	y	3	2.9971	2.9544	2.7716	2.2981	1.3852	0	M1	3.4
	y_0	y_1	y_2	y_3	y_4	y_5	y_6																			
x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1																			
y	3	2.9971	2.9544	2.7716	2.2981	1.3852	0																			
$y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6 = \text{“42.1206”}$ $\{3 + 4(2.9971 + 2.7716 + 1.3852) + 2(2.9544 + 2.2981) + 0\}$	M1	1.1b																								
Awrt 42.121	A1	1.1b																								

So volume required is approx. $\frac{85}{1000} \times \frac{1}{3} \times "42.1206" \times 2$	M1	3.1a
= awrt 0.3978 m ³	A1	3.2a
	(6)	

B1: Correct strip width for the method chosen $\frac{1}{6}$ for the interval $[0, 1]$ and later doubled.

M1: Uses the model to find the appropriate values for the method. May use that the function is even to only work out half of them, so may be implied by use in the formula. At least two correct values to 4 s.f. needed for the method.

M1: Applies the "bracket" of Simpson's rule, " $y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6$ ". Coefficients must be correct.

A1: Correct value for the "bracket". If not explicitly seen, may be implied by awrt 4.680 as a value for the cross section area following correct values.

M1: Correct full method to find the volume. E.g. multiplies their bracket by their $\frac{h}{3}$ and by 0.085.

Accept an attempt in any consistent units, so e.g. in mm³ ie $85 \times \frac{1}{3} \times "42.1206" \times 1000^2 \times 2$
(corrected from the printed mark scheme, which has "42.203" here)

A1: Correct answer in m³.

Using 6 ordinates

Max score B0 M1 M0 A0 M0 A0

	y_0	y_1	y_2	y_3	y_4	y_5
x	-1	-0.6	-0.2	0.2	0.6	1
y	0	2.53298	2.9941	2.9941	2.53298	0

B0: Incorrect strip width

M1: Uses the model to find the appropriate values for the method. At least two correct values to 4 s.f. needed for the method.

3.

Scheme	Marks	A0
$t_0 = \frac{1}{2}$ and steps are 2 months, so $h = \frac{1}{6}$ $\left(t_1 = \frac{2}{3}, t_2 = \frac{5}{6}\right)$	B1	3.3

$\left(\frac{dP}{dt}\right)_0 = \frac{540}{5000} \left(1000 - \frac{540 \times \left(\frac{1}{2} + 1\right)}{6 \times \frac{1}{2} + 5}\right) =$ $\dots \left(97.065 = \frac{19413}{200}\right)$	M1	3.4
So when $t = \frac{2}{3}$, $P_1 = 540 + \frac{1}{6} \times '97.065' = \dots$ Or starts with $'97.065' = \frac{y_1 - 540}{\frac{1}{6}}$ and rearranges to find $P_1 = \dots$	M1	1.1b
$= \frac{222471}{400} = 556.1775$	A1	1.1b
$\left(\frac{dP}{dt}\right)_1 = \frac{'556.1775'}{5000} \left(1000 - \frac{'556.1775' \times \left(\frac{2}{3} + 1\right)}{6 \times \frac{2}{3} + 5}\right) =$ $\dots (99.778 \dots)$	M1	3.4
So when $t = \frac{5}{6}$, $P_2 = '556.1775' + \frac{1}{6} \times '99.778 \dots' = \dots (572.807 \dots)$	M1	1.1b
So there are estimated to be 572 or 573 deer after 10 months.	A1	3.2a
	(7)	
	(7 marks)	

Notes

B1: Uses the given information to set up correct parameters for the model, $t_0 = \frac{1}{2}$, $\left(t_1 = \frac{2}{3}, t_2 = \frac{5}{6}\right)$ and $h = \frac{1}{6}$ seen or implied.

M1: Uses $P_0 = 540$ and their value for t_0 in the given equation to find a value for $\left(\frac{dP}{dt}\right)_0$

M1: Applies the approximation formula with 540, their h and their $\left(\frac{dP}{dt}\right)_0$ to find a value for P_1

A1: Correct approximation P at $t = \frac{2}{3}$. Accept awrt 556.2

M1: Uses $t_1 = t_0 + h$ and their P_1 in the given equation to find a value for $\left(\frac{dP}{dt}\right)_1$.

M1: Uses the approximation a second time with their h , their P_1 and their $\left(\frac{dP}{dt}\right)_1$ to find a value for P_2

A1: Correct answer. Accept either 572 or 573.

Useful table of values for reference

n	P_n	t	$\frac{dP}{dt}$	$h \frac{dP}{dt}$
0	540	$\frac{1}{2}$	97.065	16.1775
1	556.1775	$\frac{2}{3}$	99.77870698	16.6297845
2	572.807	$\frac{5}{6}$		

Note use of $t_0 = 6$ and $h = 2$ leads to

$$\left(\frac{dP}{dt}\right)_0 = \frac{100494}{1025} = 98.04 \dots \quad P_1 = 736.08 \dots \quad \left(\frac{dP}{dt}\right)_1 = 128.8 \dots$$

$P_2 = 993.7 \dots$ which scores a maximum of B0 M1 M1 A0 M1 M1 A0

4.

Scheme	Marks	AO
$\frac{d^2y}{dx^2} = 2y^2 - x - 1 \Rightarrow \left(\frac{d^2y}{dx^2}\right)_0 = 0 - 0 - 1 = -1$	B1	1.1b
$\left(\frac{dy}{dx}\right)_0 = 3 \Rightarrow \frac{(y_1 - y_{-1})}{0.2} \approx 3$	B1	1.1b
$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{(y_1 - 2y_0 + y_{-1})}{h^2} \Rightarrow \frac{y_1 - 2(0) + y_{-1}}{0.01} \approx -1$	M1	1.1b
$y_1 \approx \frac{1}{2}(0.6 - 0.01) = 0.295$	dM1	2.1
$\frac{d^2y}{dx^2} = 2y^2 - x - 1 \Rightarrow \left(\frac{d^2y}{dx^2}\right)_1 = 2(0.295)^2 - 0.1 - 1 = -0.92595$	dM1	1.1b
$\left(\frac{d^2y}{dx^2}\right)_1 \approx \frac{(y_2 - 2y_1 + y_0)}{h^2} \Rightarrow \frac{y_2 - 2(0.295) + 0}{0.01} \approx -0.92595 \Rightarrow y_2 = \dots$	dM1	2.1
$y_2 \approx 2(0.295) - 0.92595 \times 0.01 = 0.581 \text{ (3 s.f.)}$	A1	1.1b
	(7)	
	(7 marks)	

Notes

B1: Correct value for the second derivative using the differential equation

B1: Correct equation in terms of y_1 and y_{-1} using the first order approximation

M1: Uses the second order approximation to obtain another equation in terms of y_1 and y_{-1}

M1: Uses their two equations in y_1 and y_{-1} and solves together to find y at $x = 0.1$.

M1: Uses the differential equation with their y at $x = 0.1$ and $x = 0.1$ to find a value for the second derivative at $x = 0.1$

M1: Completes the process by using the second order approximation and their second derivative to obtain a value for y_2

A1: Correct value for y at $x = 0.2$

Note that all method marks are dependent