

QQQ – Further Pure 1 – Chapter 6 – Taylor Series (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. The displacement x metres of a particle at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + x + \cos x = 0$$

When $t = 0$, $x = 0$ and $\frac{dx}{dt} = \frac{1}{2}$.

Find a Taylor series solution for x in ascending powers of t , up to and including the term in t^3 .

(5)

(Total 5 marks)

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2.

- (a) Find the Taylor expansion of $\cos 2x$ in ascending powers of $\left(x - \frac{\pi}{4}\right)$ up to and including the term in $\left(x - \frac{\pi}{4}\right)^5$. **(5)**

- (b) Use your answer to (a) to obtain an estimate of $\cos 2$, giving your answer to 6 decimal places. **(3)**

(Total 8 marks)

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3.

$$y = \ln(e^{2x} \cos 3x) \quad -\frac{1}{2} < x < \frac{1}{2}$$

(a) Show that

$$\frac{dy}{dx} = 2 - 3 \tan 3x \quad (2)$$

(b) Determine $\frac{d^4y}{dx^4}$ **(3)**

(c) Hence determine the first 3 non-zero terms in ascending powers of x of the Maclaurin series expansion of $\ln(e^{2x} \cos 3x)$, giving each coefficient in simplest form. **(3)**

(d) Use the Maclaurin series expansion for $\ln(1 + x)$ to write down the first 4 non-zero terms in ascending powers of x of the Maclaurin series expansion of $\ln(1 + kx)$, where k is a constant. **(1)**

(e) Hence determine the value of k for which

$$\lim_{x \rightarrow 0} \left(\frac{1}{x^2} \ln \frac{e^{2x} \cos 3x}{1 + kx} \right)$$

exists. **(3)**

(Total 12 marks)

Solutions

1.

Scheme	Marks
$f'r'(t) = -x - \cos x, \quad f'r'(0) = -1$	B1
$f''r'(t) = (-1 + \sin x)\frac{dx}{dt}, \quad f''r'(0) = -0.5$	M1A1
$f(t) = f(0) + tf'r'(0) + \frac{t^2}{2}f'r'(0) + \frac{t^3}{3!}f''r'(0) + \dots$ $= 0.5t - 0.5t^2 - \frac{1}{12}t^3 + \dots$	M1 A1
	(5 marks)

2.

Mark scheme (a)

Scheme	Marks
$f(x) = \cos 2x, \quad f\left(\frac{\pi}{4}\right) = 0$	
$f'(x) = -2 \sin 2x, \quad f'\left(\frac{\pi}{4}\right) = -2$	M1
$f''(x) = -4 \cos 2x, \quad f''\left(\frac{\pi}{4}\right) = 0$	
$f'''(x) = 8 \sin 2x, \quad f'''\left(\frac{\pi}{4}\right) = 8$	A1
$f^{(iv)}(x) = 16 \cos 2x, \quad f^{(iv)}\left(\frac{\pi}{4}\right) = 0$	
$f^{(v)}(x) = -32 \sin 2x, \quad f^{(v)}\left(\frac{\pi}{4}\right) = -32$	A1
$\cos 2x = f\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right) + \frac{f''\left(\frac{\pi}{4}\right)}{2}\left(x - \frac{\pi}{4}\right)^2 +$ $\frac{f'''\left(\frac{\pi}{4}\right)}{3!}\left(x - \frac{\pi}{4}\right)^3 + \dots$ Three terms are sufficient to establish method	M1
$\cos 2x = -2\left(x - \frac{\pi}{4}\right) + \frac{4}{3}\left(x - \frac{\pi}{4}\right)^3 - \frac{4}{15}\left(x - \frac{\pi}{4}\right)^5 + \dots$	A1
	(5)

Notes

(corrected from the printed mark scheme: the $(x - \frac{\pi}{4})^3$ term of the general expansion is printed with $f'(\frac{\pi}{4})$ instead of $f'''(\frac{\pi}{4})$; also, $f^{(v)}(x)$ is printed as $32 \sin 2x$, but differentiating $16 \cos 2x$ gives $-32 \sin 2x$, which matches the value -32 at $\frac{\pi}{4}$)

Mark scheme (b)

Scheme	Marks
Substitute $x = 1 - \frac{\pi}{4} \approx 0.21460$	B1
$\cos 2 = -2(x - \frac{\pi}{4}) + \frac{4}{3}(x - \frac{\pi}{4})^3 - \frac{4}{15}(x - \frac{\pi}{4})^5 + \dots$ ≈ -0.416147 cao	M1 A1
	(3)
	(8 marks)

3.

Mark scheme (a)

Scheme	Marks	AO
$y = \ln(e^{2x} \cos 3x) \Rightarrow \frac{dy}{dx} = \frac{2e^{2x} \cos 3x - 3e^{2x} \sin 3x}{e^{2x} \cos 3x}$ or $y = \ln e^{2x} + \ln \cos 3x = 2x + \ln \cos 3x \Rightarrow \frac{dy}{dx} = 2 + \frac{-3 \sin 3x}{\cos 3x}$	M1	2.1
$= 2 - 3 \tan 3x *$	A1*	1.1b
	(2)	

Notes

M1: Attempts differentiation of the given function by applying both the chain rule and the product rule. Look for the correct form but they may make slips with signs or coefficients. Alternatively, applies the sum law for logs and differentiates using the chain rule.

A1*: Correct proof.

Mark scheme (b)

Scheme	Marks	AO
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$\frac{d^2y}{dx^2} = -9 \sec^2 3x$	B1	1.1b
$\frac{d^3y}{dx^3} = -54 \sec^2 3x \tan 3x$	M1 A1	2.1 1.1b
$\Rightarrow \frac{d^4y}{dx^4} = -162 \sec^4 3x - 324 \sec^2 3x \tan^2 3x$ (oe)		
	(3)	

Notes

B1: Correct second derivative.

M1: Continues the differentiation using the chain rule and product rule to reach the 4th derivative. Look for correct forms, allowing for slips in signs or coefficients only when differentiating. May be given in terms of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ etc for this mark.

A1: Correct 4th derivative in terms of x . Accept alternatives. Like terms should be gathered and coefficients simplified but isw after a correct suitable answer.

Mark scheme (c)

Scheme	Marks	AO
$(y)_0 = 0, \left(\frac{dy}{dx}\right)_0 = 2, \left(\frac{d^2y}{dx^2}\right)_0 = -9, \left(\frac{d^3y}{dx^3}\right)_0 = 0, \left(\frac{d^4y}{dx^4}\right)_0 = -162$	M1	1.1b
$y = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \dots$	M1	2.1
$(y =) 0 + 2x - \frac{9x^2}{2} - \frac{162x^4}{24} + \dots = 2x - \frac{9x^2}{2} - \frac{27x^4}{4}$	Alcso	1.1b
	(3)	

Notes

M1: Attempts the values of the derivatives at $x = 0$ up to at least the third derivative. If substitution not seen at least two values should be correct for their derivatives, with values for others.

M1: Substitutes the values of their derivatives into the correct Maclaurin expansion formula up to at least the third non-zero value for their derivatives.

A1cso: Must have obtained a correct 4th derivative in (b) (though need not have been in terms of x).
 Correct expansion with simplified coefficients. May be missing the “ $y =$ ” and allow “ $f(x) =$ ”. Note the correct series will be obtained from various incorrect 4th derivatives due to some terms evaluating to 0, but use of a clearly incorrect 4th derivative score A0 here. However, an initially correct 4th derivative, with incorrect simplification, can score A1 bod if no incorrect work is shown (ie correct values stated with no substitution shown, as they may have used the initially correct answer).

Mark scheme (d)

Scheme	Marks	AO
$\ln(1 + kx) = kx - \frac{k^2 x^2}{2} + \frac{k^3 x^3}{3} - \frac{k^4 x^4}{4} + \dots$	B1	2.2a
	(1)	

Notes

B1: Deduces the correct expansion in any form.

Mark scheme (e)

Scheme	Marks	AO
$\ln \frac{e^{2x} \cos 3x}{1 + kx} = \ln(e^{2x} \cos 3x) - \ln(1 + kx)$ $= \ln(e^{2x} \cos 3x) - \ln(1 + kx)$ $= 2x - \frac{9x^2}{2} - \frac{27x^4}{4} + \dots - \left(kx - \frac{k^2 x^2}{2} + \dots \right)$	M1	3.1a
$= \frac{1}{x^2} \left((2 - k)x - \frac{(9 - k^2)x^2}{2} - \frac{k^3 x^3}{3} - \frac{(27 - k^4)x^4}{4} + \dots \right)$ For the limit to exist $2 - k = 0 \Rightarrow k = \dots$	M1	3.1a
$k = 2$	A1	2.2a
	(3)	
	(12 marks)	

Notes

M1: Applies the subtraction law of logarithms and then substitutes their expansions.

M1: Realises that the x terms will determine the required value of k and so collects the x terms and sets the coefficient to zero and solves for k (may be implied).

A1: For $k = 2$