

QQQ – Further Pure 1 – Chapter 5 – The t-formulae (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1.

- (a) Given that $t = \tan \frac{x}{2}$ prove that

$$\cos x \equiv \frac{1 - t^2}{1 + t^2}$$

(3)

- (b) Show that the equation

$$3 \tan x - 10 \cos x = 10$$

can be written in the form

$$(t + 2)(at^2 + bt + c) = 0$$

where $t = \tan \frac{x}{2}$ and a, b and c are integers to be determined.

(4)

- (c) Hence solve, for $-180^\circ < x < 180^\circ$, the equation

$$3 \tan x - 10 \cos x = 10$$

(5)

(Total 12 marks)

2.

- (i) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to prove that

$$\cot x + \tan\left(\frac{x}{2}\right) = \operatorname{cosec} x \quad x \neq n\pi, n \in \mathbb{Z}$$

(2)

(ii)

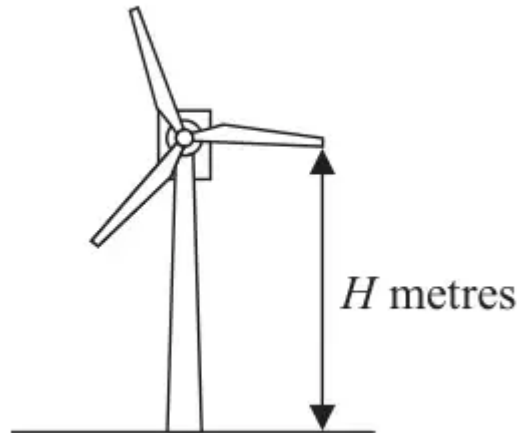


Figure 1

An engineer models the vertical height above the ground of the tip of one blade of a wind turbine, shown in Figure 1. The ground is assumed to be horizontal.

The vertical height of the tip of the blade above the ground, H metres, at time x seconds after the wind turbine has reached its constant operating speed, is modelled by the equation

$$H = 90 - 30 \cos(120x)^\circ - 40 \sin(120x)^\circ \quad (\text{I})$$

- (a) Show that $H = 60$ when $x = 0$ (1)

Using the substitution $t = \tan(60x)^\circ$

- (b) show that equation (I) can be rewritten as

$$H = \frac{120t^2 - 80t + 60}{1 + t^2}$$

(3)

- (c) Hence find, according to the model, the value of x when the tip of the blade is 100 m above the ground for the first time after the wind turbine has reached its constant operating speed. (5)

(Total 11 marks)

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