

QQQ – Further Pure 1 – Chapter 4 – Inequalities (Version 1)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1.

- (a) Use algebra to determine the values of x for which

$$\frac{5x}{x-2} \geq 12$$

(4)

- (b) Hence, given that x is an integer, deduce the value of x .

(1)

(Total 5 marks)

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2.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

- (a) Sketch the graph of the curve with equation

$$y = \frac{1}{x^2}$$

(2)

- (b) Solve, using algebra, the inequality

$$3 - 2x^2 > \frac{1}{x^2}$$

(5)

(Total 7 marks)

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3. A physics student is studying the movement of particles in an electric field. In one experiment, the distances in micrometres of two moving particles, A and B , from a fixed point O are modelled by

$$d_A = |5t - 31|$$

$$d_B = |3t^2 - 25t + 8|$$

respectively, where t is the time in seconds after motion begins.

- (a)** Use algebra to find the range of time for which particle A is further away from O than particle B is from O . **(8)**

It was recorded that the distance of particle B from O was less than the distance of particle A from O for approximately 4 seconds.

- (b)** Use this information to assess the validity of the model. **(2)**

(Total 10 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
E.g. $(x \neq 2 \Rightarrow) 5x = 12(x - 2) \Rightarrow x = \dots$ $\frac{5x}{x - 2} \geq 12 \Rightarrow 5x(x - 2) \geq 12(x - 2)^2 \Rightarrow (x - 2)(5x - 12(x - 2)) \geq 0 \Rightarrow x = \dots$ or $\frac{5x}{x - 2} \geq 12 \Rightarrow \frac{5x - 12(x - 2)}{x - 2} \geq 0 \Rightarrow x = \dots$ o.e.	M1	2.1
E.g. $x = 2$ and $(x \neq 2 \Rightarrow) 5x = 12(x - 2) \Rightarrow x = \dots$ $(x - 2)(-7x + 24) \geq 0$ or $(x - 2)(7x - 24) \leq 0 \Rightarrow x = \dots, \dots$ or $7x^2 - 38x + 48 \leq 0$ or $-7x^2 + 38x - 48 \geq 0 \Rightarrow x = \dots, \dots$ or $\frac{24 - 7x}{x - 2} \geq 0 \Rightarrow x = \dots, \dots$	dM1	1.1b
Critical values $x = 2, \frac{24}{7}$	A1	1.1b
$2 < x \leq \frac{24}{7}$	A1	2.3
	(4)	

Notes

M1: For an algebraic method to find the critical value aside from $x = 2$. May set equal and use $x \neq 2$ or $x < 2$ and $x > 2$ to find the x coordinate of the intersection of line and curve, or may multiply through by $(x - 2)^2$ and gather terms onto one side and solve the quadratic, or gather all terms onto one side and put over a common denominator and solve the quadratic. Allow with any inequality or equality for the first two marks.

dM1: Finds **both** the critical values by a correct algebraic method (allowing for slips rearranging). The $x = 2$ may just be stated or used as a boundary value of the interval. Use of quadratic formula - usual rules. Dependent on previous method mark.

A1: Correct critical values stated or used in solution. Allow awrt 3.43 for $\frac{24}{7}$ for this mark. A0 if other values also used.

A1: Deduces the correct inequality. Must be exact. Accept alternative notations, such as $\left(2, \frac{24}{7}\right]$ but formal set notation is not required - score the inequality given. Accept as separate inequalities.

If $2 < x \leq \frac{24}{7}$ follows $2 \leq x \leq \frac{24}{7}$ arising from $(x - 2)(7x - 24) \leq 0$ (oe) then allow A1 b.o.d. that the latter answer is a rejection of the 2 being included.

(Brackets that are missing from the printed mark scheme because of a font fault have been restored.)

Mark scheme (b)

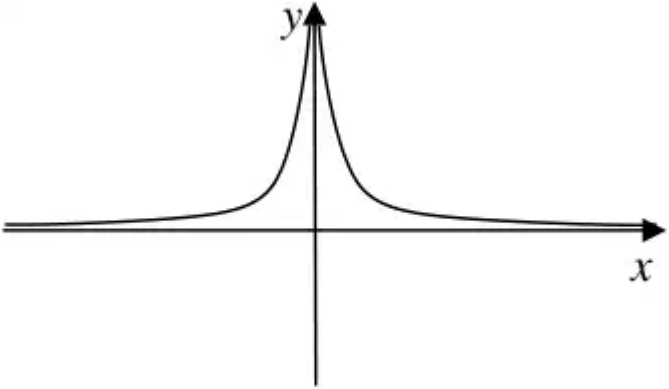
Scheme	Marks	AO
$x = 3$	B1	2.2a
	(1)	
	(5 marks)	

Notes

B1cao: Deduces the correct value for x and no other values as long as 3 is in their solution set from (a). Allow if the endpoint 2 was included in their answer to (a) as long as it is not given as a solution for (b).

2.

Mark scheme (a)

Scheme	Marks	AO
 One branch correct. See notes.	M1	1.1b
See notes	A1	1.1b

Notes

M1: For one branch with the correct shape in quadrant 1 or quadrant 2 (probably first quadrant)

Give tolerance on curves bending away from the axis where it is clear the correct shape is meant provided there is no clear intention to draw a minimum.

Ignore any dashed lines/scale and just look for the shape.

The branch must not clearly intentionally meet or cross either axis.

Condone gaps between the branches and the axes as long as the asymptotic behaviour is intended.

A1: Fully correct sketch. Branches should show asymptotic behaviour to the axes but condone gaps as above for this mark, clear bending away is A0.

Ignore dotted lines at or near either axis if they are clearly intended to indicate they are asymptotes.

Mark scheme (b)

Scheme	Marks	AO
$3 - 2x^2 = \frac{1}{x^2} \Rightarrow 2x^4 - 3x^2 + 1 = 0 \Rightarrow x^2 = \dots$ or e.g. $3 - 2x^2 = \frac{1}{x^2} \Rightarrow 3 - 2x^2 - \frac{1}{x^2} = 0 \Rightarrow \frac{3x^2 - 2x^4 - 1}{x^2} = 0 \Rightarrow x^2 = \dots$	M1	1.1b
$x^2 = 1, \frac{1}{2}$ or e.g. $2x^4 - 3x^2 + 1 = 0 \Rightarrow (2x^2 - 1)(x^2 - 1) = 0 \Rightarrow x^2 = 1, \frac{1}{2}$ or e.g. $-2x^4 + 3x^2 - 1 = 0 \Rightarrow (-2x^2 + 1)(x^2 - 1) = 0 \Rightarrow (x - 1)(x + 1)(-\sqrt{2}x + 1)(\sqrt{2}x + 1) = 0$ $\Rightarrow x = \pm 1, \pm \frac{1}{\sqrt{2}}$	A1	1.1b
$x = \pm 1, \pm \frac{\sqrt{2}}{2}$	B1	1.1b
$-1 < x < -\frac{\sqrt{2}}{2}, \quad \frac{\sqrt{2}}{2} < x < 1$	M1 A1	2.1 2.2a
	(5)	
	(7 marks)	

Notes

The question says all working to be shown and solutions not entirely from a calculator.

M1: For an algebraic method to find the critical values so requires:

- e.g. multiplies both sides by x^2 and collects terms to one side or collects terms to one side and puts over a common denominator
- solves the resulting 3 term equation for x^2 (usually $2x^4 - 3x^2 + 1 = 0$). This can be by any method e.g. factors, formula, completing the square or calculator. The usual rules apply so if values are just written down, they must be correct for their equation.

A1: For $x^2 = 1$, $\frac{1}{2}$ (ignore any reference to $x = 0$) or if using factors may proceed directly to x

B1 (A1 in Epen): For all four correct and exact cv's and no others apart from $x = 0$

M1: Forms regions of the correct form using their **four** critical values.

Must have four **non-zero** cv's a, b, c, d where $a < b < c < d$ and form 2 "inside" inequalities with the cv's in ascending order e.g. $a < x < b$, $c < x < d$

The directions must be correct but allow strict or non-strict inequalities or a mixture of both.

Accept alternative notation including set notation e.g.

$$\left\{x \in \mathbb{R} : -1 < x < -\frac{\sqrt{2}}{2}\right\}, \left\{x \in \mathbb{R} : \frac{\sqrt{2}}{2} < x < 1\right\} \text{ or e.g. } \left(-1, -\frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, 1\right)$$

There must be no other regions.

A1: Correct regions only. Accept with $\frac{1}{\sqrt{2}}$ or e.g. $\sqrt{\frac{1}{2}}$ (must be exact).

Accept equivalent correct notation but if using set notation do not condone $\cap$

Special case:

Candidates who solve the quartic inequality using a calculator e.g.

$$3 - 2x^2 > \frac{1}{x^2} \Rightarrow 2x^4 - 3x^2 + 1 < 0 \Rightarrow -1 < x < -\frac{\sqrt{2}}{2}, \quad \frac{\sqrt{2}}{2} < x < 1$$

Score a maximum of 1 mark - score as M0A0B0M1A0 in EPEN

(Corrected from the printed mark scheme: the special case prints $2x^4 - 3x^2 + 1 > 0$; multiplying by x^2 gives $2x^4 - 3x^2 + 1 < 0$.)

3.

Mark scheme (a)

Scheme	Marks	AO
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Establishes need for $ 5t - 31 > 3t^2 - 25t + 8 $ and attempts to find all C.V.'s to form the critical region, e.g. via a sketch.	M1	3.1a
$5t - 31 = 3t^2 - 25t + 8 \Rightarrow 3t^2 - 30t + 39 = 0 \Rightarrow t = \dots$ $t = 5 \pm 2\sqrt{3}$	M1 A1	1.1b 3.4
$-(5t - 31) = 3t^2 - 25t + 8 \Rightarrow 3t^2 - 20t - 23 = 0 \Rightarrow t = \dots$ $t = (-1), \frac{23}{3}$	M1 A1	2.1 3.4
Selects “insides” $(-1 <) \alpha < t < \beta, \gamma < t < \delta$ where $\alpha < \beta < \gamma < \delta$	M1	2.2a
$(-1 <) 0 \leq t < 5 - 2\sqrt{3}$ or $\frac{23}{3} < t < 5 + 2\sqrt{3}$	A1	1.1b
Both regions, $0 \leq t < 5 - 2\sqrt{3}$ and $\frac{23}{3} < t < 5 + 2\sqrt{3}$	A1	2.3
	(8)	

Notes

M1: Sets problem up as an inequalities problem, and forms complete strategy to solve – must see attempt at all critical values and some attempt to form at least one range from them. May be scored if algebra not used. This mark is for showing an overall awareness of the problem.

M1: Attempts to find C.V.'s for the “positives”. Any valid method using algebra. Must see an attempt to find a 3TQ (oe), but allow answers from calculator once a $3TQ = 0$ is seen.

A1: Correct C.V.'s, both required.

M1: Attempts to find the other C.V.'s (same conditions as above)

A1: Correct C.V.'s. Need not see the negative value stated as $t > 0$ is required. (If both given, they must be correct)

M1: Selects correct critical regions, shows the idea the “insides” are needed.

$(-1 <) \alpha < t < \beta, \gamma < t < \delta$ where $\alpha < \beta < \gamma < \delta$ are their four critical values, possibly truncated at 0 as long as no more than 1 is negative.

A1: One correct interval. Allow with loose or strict inequalities. Allow this mark if $-1 < t < 5 - 2\sqrt{3}$ is given. Allow any variable for this mark.

A1: Fully correct solution. Must start at zero for the leftmost interval but accept $<$ or $\leq$ here. Must be using t

Alternative

Scheme	Marks	AO
Establishes need for $ 5t - 31 > 3t^2 - 25t + 8 $ and attempts to find all C.V.'s to form the critical region, e.g. via a sketch.	M1	3.1a
$(5t - 31)^2 = (3t^2 - 25t + 8)^2 \Rightarrow \dots$ $9t^4 - 150t^3 + 648t^2 - 90t - 897 = 0$	M1 A1	1.1b 3.4
Solves $9t^4 - 150t^3 + 648t^2 - 90t - 897 = 0 \Rightarrow t = \dots$ $t = 5 \pm 2\sqrt{3}, \frac{23}{3}, \{-1\}$	M1 A1	2.1 3.4
Selects "insides" $(-1 <) \alpha < t < \beta, \gamma < t < \delta$ where $\alpha < \beta < \gamma < \delta$	M1	2.2a
$(-1 <) 0 \leq t < 5 - 2\sqrt{3}$ or $\frac{23}{3} < t < 5 + 2\sqrt{3}$ condone any variable	A1	1.1b
Both regions, $0 \leq t < 5 - 2\sqrt{3}$ and $\frac{23}{3} < t < 5 + 2\sqrt{3}$ must be using t	A1	2.3
	(8)	

M1: Sets problem up as an inequalities problem, and forms complete strategy to solve – must see attempt at all critical values and some attempt to form at least one ranges from them. May be scored if algebra not used. This mark is for showing an overall awareness of the problem.

M1: Attempts to find C.V.'s by squaring both sides and forming a quartic equation.

A1: Correct quartic equation

M1: Attempts to solve their quartic equation

A1: All 4 correct exact C.V.'s. Need not see the negative value stated as $t > 0$ is required.

M1: Selects correct critical regions, shows the idea the "insides" are needed.

$(-1 <) \alpha < t < \beta, \gamma < t < \delta$ where $\alpha < \beta < \gamma < \delta$ are their four critical values, possibly truncated at 0 as long as no more than 1 is negative.

A1: One correct interval. Allow with loose or strict inequalities. Allow this mark if $-1 < t < 5 - 2\sqrt{3}$ is given. Allow any variable for this mark.

A1: Fully correct solution. Must start at zero for the leftmost interval but accept $<$ or $\leq$ here. Must be using t

Mark scheme (b)

Scheme	Marks	AO
Time that B is closer to O than particle A is $5 + 2\sqrt{3} - \frac{23}{3} + 5 - 2\sqrt{3} = \frac{7}{3}$ seconds.	M1	3.4

This is considerably less than 4 seconds so the model does not seem appropriate.	A1ft	3.5a
	(2)	
	(10 marks)	

Notes

M1: Uses their result from (a) to determine how long particle B is closer to O than particle A is.

A1: Draws a suitable conclusion for their answer to (a) – if correct in (a) it is that the model is not very suitable. Must not have a negative time.