

QQQ – Further Pure 1 – Chapter 1 – Vectors (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. Vectors $\mathbf{u}$ and $\mathbf{v}$ are given by

$$\mathbf{u} = 5\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \mathbf{v} = a\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$$

where a is a constant.

(a) Determine, in terms of a , the vector product $\mathbf{u} \times \mathbf{v}$ (2)

Given that

- $\overrightarrow{AB} = 2\mathbf{u}$
- $\overrightarrow{AC} = \mathbf{v}$
- the area of triangle ABC is 15

(b) determine the possible values of a . (4)

(Total 6 marks)

2.

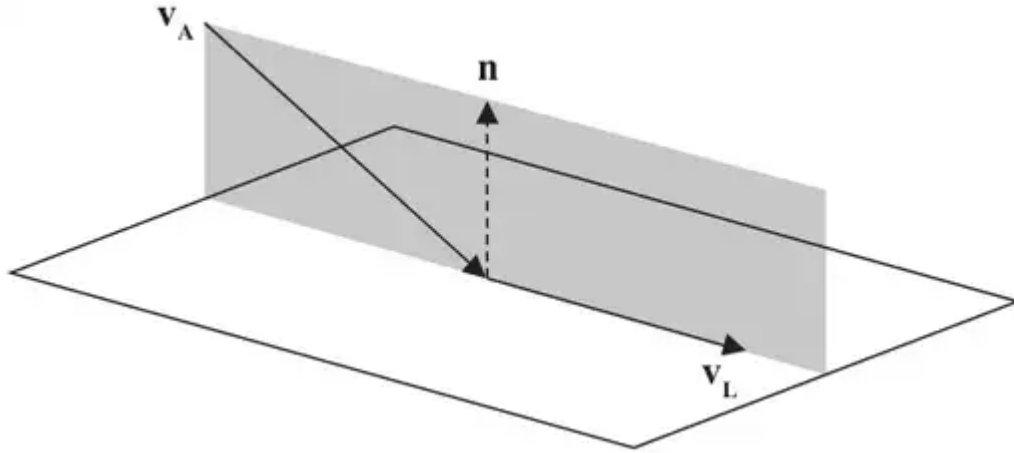


Figure 2

A small aircraft is landing in a field.

In a model for the landing the aircraft travels in different straight lines before and after it lands, as shown in Figure 2.

The vector $\mathbf{v}_A$ is in the direction of travel of the aircraft as it approaches the field.

The vector $\mathbf{v}_L$ is in the direction of travel of the aircraft after it lands.

With respect to a fixed origin, the field is modelled as the plane with equation

$$x - 2y + 25z = 0$$

and

$$\mathbf{v}_A = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$$

(a) Write down a vector $\mathbf{n}$ that is a normal vector to the field. (1)

(b) Show that $\mathbf{n} \times \mathbf{v}_A = \lambda \begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix}$, where λ is a constant to be determined. (2)

When the aircraft lands it remains in contact with the field and travels in the direction $\mathbf{v}_L$

The vector $\mathbf{v}_L$ is in the same plane as both $\mathbf{v}_A$ and $\mathbf{n}$ as shown in Figure 2.

(c) Determine a vector which has the same direction as $\mathbf{v}_L$ (3)

(d) State a limitation of the model. (1)

(Total 7 marks)

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3. The parallelogram P has vertices E , F , G and H , with coordinates

$$E(10, -1, -6) \quad F(7, -2, 7) \quad G(10, 2, 9) \quad H(13, 3, -4)$$

(a) Determine the exact area of P , giving your answer as a simplified surd. **(3)**

The line l passes through E and is perpendicular to the plane containing P

(b) Determine an equation for l giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors. **(2)**

Given that

- P is one face of a parallelepiped
 - the opposite face of the parallelepiped lies in the plane Π
 - the point $(25, -4, 13)$ lies in Π
- (c)** determine the volume of the parallelepiped. **(5)**

(Total 10 marks)

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Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 4 & -3 \\ a & -6 & 2 \end{vmatrix} = \dots \mathbf{i} - \dots \mathbf{j} + \dots \mathbf{k}$	M1	1.1b
$= -10\mathbf{i} - (10 + 3a)\mathbf{j} - (30 + 4a)\mathbf{k} \text{ or e.g. } \begin{pmatrix} -10 \\ -10 - 3a \\ -30 - 4a \end{pmatrix}$	A1	1.1b
	(2)	

Notes

M1: Evidence of a correct method for the vector product. May be implied by two out of three correct components if no method shown or by correct work for 1 component.

Attempting $\mathbf{v} \times \mathbf{u}$ scores M0

A1: Correct answer.

Allow equivalent expressions and condone e.g. $-10\mathbf{I} + (-10 - 3a)\mathbf{J} + (-30 - 4a)\mathbf{K}$

Award the mark once a correct vector is seen and isw if necessary.

Must be a vector not coordinates.

Mark scheme (b)

Scheme	Marks	AO
$\text{Area} = \frac{1}{2} 2\mathbf{u} \times \mathbf{v} = 15 \Rightarrow$ $ -10\mathbf{i} - (10 + 3a)\mathbf{j} - (30 + 4a)\mathbf{k} = 15$ Or e.g. $\text{Area} = \frac{1}{2} 2\mathbf{u} \times \mathbf{v} = \frac{1}{2} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & 8 & -6 \\ a & -6 & 2 \end{vmatrix} =$ $\frac{1}{2} \left \begin{pmatrix} -20 \\ -20 - 6a \\ -60 - 8a \end{pmatrix} \right = 15$	M1	3.1a
$\Rightarrow 10^2 + (10 + 3a)^2 + (30 + 4a)^2 = 15^2$ $\Rightarrow 25a^2 + 300a + 875 = 0 (\Rightarrow a^2 + 12a + 35 = 0)$	M1	1.1b

or e.g. $20^2 + (20 + 6a)^2 + (60 + 8a)^2 = 30^2$ $\Rightarrow 400 + 400 + 240a + 36a^2 + 3600 + 960a + 64a^2 = 30^2$ $\Rightarrow 100a^2 + 1200a + 3500 = 0 \ (\Rightarrow a^2 + 12a + 35 = 0)$		
$\Rightarrow (a + 5)(a + 7) = 0 \Rightarrow a = \dots$	ddM1	1.1b
$a = -7$ or -5	A1 cso	2.2a
	(4)	
	(6 marks)	

Notes

M1: Applies a correct method for the area of the triangle and scalar multiple property of vector product to set up a vector equation in a using the vector product.

The “1/2” must be seen or implied.

This may be implied by later work e.g. $\frac{1}{2}\sqrt{20^2 + (20 + 6a)^2 + (60 + 8a)^2} = 15$

M1: Applies the modulus correctly and expands correctly to reach a quadratic in a .

Note that $\frac{1}{4}(20^2 + (20 + 6a)^2 + (60 + 8a)^2) = 15$ scores M0 (must square both sides)

ddM1: Solves their quadratic by any suitable means including a calculator to obtain at least one real root.

Depends on both previous method marks.

A1cso: Both correct values following correct work e.g. do not condone sign errors in the vector product if they fortuitously lead to the correct answers.

Note there may be more convoluted methods for the area e.g. using “ $\frac{1}{2}ab \sin C$ ” and the scalar product:

$$\frac{1}{2} |\vec{AB}| |\vec{AC}| \sin A = 15 \Rightarrow \frac{1}{2} 10\sqrt{2}\sqrt{a^2 + 40} \sin A = 15$$

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos A \Rightarrow 5a - 30 = 5\sqrt{2}\sqrt{a^2 + 40} \cos A$$

$$\Rightarrow \sin A = \sqrt{\frac{a^2 + 12a + 44}{2(a^2 + 40)}} \Rightarrow 5\sqrt{2}\sqrt{a^2 + 40} \sqrt{\frac{a^2 + 12a + 44}{2(a^2 + 40)}} = 15$$

Score M1 for a complete correct method to obtain an equation in a only then e.g.

$$5\sqrt{a^2 + 12a + 44} = 15 \Rightarrow a^2 + 12a + 35 = 0 \text{ etc.}$$

M2 for simplifying to obtain a quadratic in a .

Then as above. Use review if necessary.

Mark scheme (a)

Scheme	Marks	AO
$\mathbf{n} = \begin{pmatrix} 1 \\ -2 \\ 25 \end{pmatrix}$ or any non-zero scalar multiple thereof	B1	1.2
	(1)	

Notes

B1: Correct normal vector (any non-zero scalar multiple thereof is fine)

Mark scheme (b)

Scheme	Marks	AO
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 25 \\ 3 & -2 & -1 \end{vmatrix} = \begin{pmatrix} (-2)(-1) - (-2)(25) \\ -((1)(-1) - (3)(25)) \\ (1)(-2) - (3)(-2) \end{pmatrix} = \dots$	M1	1.1b
$= \begin{pmatrix} 52 \\ 76 \\ 4 \end{pmatrix} = 4 \begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix}$ (or correct multiple for their normal vector used)	A1	2.1
	(2)	

Notes

M1: Uses their $\mathbf{n}$ and $\mathbf{v}_A$ in cross product formula. Allow slips in coordinates as long as the intent is clear.

A1: Correct work leading to a multiple of the required vector – may be a different multiple to the one shown if their $\mathbf{n}$ was different.

Mark scheme (c)

Scheme	Marks	AO
Landing direction is perpendicular to $\mathbf{n} \times \mathbf{v}_A$ and $\mathbf{n}$ so required direction is given by $\begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 25 \end{pmatrix} = \dots$ Alternatively recognises the landing direction is the line of intersection of the plane containing $\mathbf{n}$ and $\mathbf{v}_A$ and the plane representing the field.	M1	3.1b

Finds the equation of the plane containing $\mathbf{n}$ and $\mathbf{v}_A$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix}$ $x - 2y + 25z = 0 \text{ and } 13x + 19y + z = 0$		
$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 13 & 19 & 1 \\ 1 & -2 & 25 \end{vmatrix} = \begin{pmatrix} (19)(25) - (-2)(1) \\ -((13)(25) - (1)(1)) \\ (13)(-2) - (1)(19) \end{pmatrix} = \dots$ Alternative Selects a value for either x, y or z and solves simultaneously e.g $z = -5$ leading to $x - 2y = 125$ and $13x + 19y = 5 \Rightarrow x = \dots, y = \dots$	M1	3.4
$= \begin{pmatrix} 477 \\ -324 \\ -45 \end{pmatrix} \text{ or any positive multiple thereof, e.g } \begin{pmatrix} 53 \\ -36 \\ -5 \end{pmatrix} \text{ or}$ $\begin{pmatrix} 1908 \\ -1296 \\ -180 \end{pmatrix}$	A1	1.1b
	(3)	

Notes

M1: Uses a correct strategy to find the direction, ie realises $\mathbf{v}_L$ must be perpendicular to both the vector from (b) and $\mathbf{n}$ (corrected from the printed mark scheme, which has $\mathbf{v}_A$ here). Allow for vectors used either way round.

Other methods may be possible – e.g finds the plane containing $\mathbf{n}$ and $\mathbf{v}_A$ and solves with the plane representing the field.

M1: Uses their answer to (b) with the normal vector to find a vector in the direction required. Allow for vectors either way round.

Alternative find the line of intersection of the plane containing $\mathbf{n}$ and $\mathbf{v}_A$ and the field.

A1: A correct direction vector, as shown or any positive multiple. For this mark the direction should be correct – so order of vectors must have been correct, or adapted to correct direction if initially incorrect.

(The equation of the plane containing $\mathbf{n}$ and $\mathbf{v}_A$ is garbled in the printed mark scheme; it is shown here

as $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 19 \\ 1 \end{pmatrix}$, which gives the equation $13x + 19y + z = 0$ that follows it.)

Mark scheme (d)

Scheme	Marks	AO
Any acceptable reason e.g. • Paths would not be linear • May have some lateral movement • Could be affected by cross winds • Field might not be flat	B1	3.5b
	(1)	
	(7 marks)	

Notes

B1: Any correct limitation given. See scheme for examples.

3.

Mark scheme (a)

Scheme	Marks	AO
e.g. $\overrightarrow{EF} = (7 - 10)\mathbf{i} + (-2 + 1)\mathbf{j} + (7 + 6)\mathbf{k}$ $\overrightarrow{EH} = (13 - 10)\mathbf{i} + (3 + 1)\mathbf{j} + (-4 + 6)\mathbf{k}$	M1	3.1a
For example, $ \overrightarrow{EF} \times \overrightarrow{EH} = \left\ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -1 & 13 \\ 3 & 4 & 2 \end{vmatrix} \right\ = -54\mathbf{i} + 45\mathbf{j} - 9\mathbf{k} = \sqrt{54^2 + 45^2 + 9^2}$	M1	2.1
$9\sqrt{62} \text{ cso}$	A1	1.1b
	(3)	

Notes

M1: Adopts a correct strategy by finding 2 appropriate vectors that will enable the area to be calculated, not two parallel vectors. If no method seen then 2 correct values will imply this mark.

$$\overrightarrow{EF} = \overrightarrow{HG} = \begin{pmatrix} -3 \\ -1 \\ 13 \end{pmatrix}, \overrightarrow{EH} = \overrightarrow{FG} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}, \overrightarrow{EG} = \begin{pmatrix} 0 \\ 3 \\ 15 \end{pmatrix}, \overrightarrow{FH} = \begin{pmatrix} 6 \\ 5 \\ -11 \end{pmatrix},$$

M1: Forms the cross product of 2 non-parallel vectors and calculates the magnitude to find that required area. May split into two triangles and then add the areas. If uses the cross product of the diagonal vectors they would need to divide their answer by 2 to score this mark.

A1: Correct area cso, look out for correct signs

Mark scheme (b)

Scheme	Marks	AO
$(\mathbf{r} - (10\mathbf{i} - \mathbf{j} - 6\mathbf{k})) \times (6\mathbf{i} - 5\mathbf{j} + \mathbf{k}) = \mathbf{0}$ or $\left(\mathbf{r} - \begin{pmatrix} 10 \\ -1 \\ -6 \end{pmatrix} \right) \times \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix} = \mathbf{0}$ o.e. Or $(\mathbf{r} - (10\mathbf{i} - \mathbf{j} - 6\mathbf{k})) \times (54\mathbf{i} - 45\mathbf{j} + 9\mathbf{k}) = \mathbf{0}$ or $\left(\mathbf{r} - \begin{pmatrix} 10 \\ -1 \\ -6 \end{pmatrix} \right) \times \begin{pmatrix} 54 \\ -45 \\ 9 \end{pmatrix} = \mathbf{0}$ o.e.	M1 A1ft	1.1b 2.2a
	(2)	

Notes

M1: Substitutes the position and direction vectors into the correct positions, condone the lack of brackets, and a sign slip and missing = 0

A1ft: Deduces the correct equation in the required form, follow through on their normal vector as long as both method marks scored in (a), must have correct bracketing and = 0

Mark scheme (c)

Scheme	Marks	AO
$6x - 5y + z = d \rightarrow d = 6 \times 25 - 5 \times (-4) + 13$ $54x - 45y + 9z = d \rightarrow d = 54 \times 25 - 45 \times (-4) + 9 \times 13$	M1	3.1a
$6x - 5y + z = 183$ o.e. $54x - 45y + 9z = 1647$	A1	1.1b
$6(10 + 6\lambda) - 5(-1 - 5\lambda) + \lambda - 6 = 183 \Rightarrow \lambda = \dots (2)$ $54(10 + 6\lambda) - 45(-1 - 5\lambda) + 9(\lambda - 6) = 1647 \Rightarrow \lambda = \dots (2)$ Or	M1	3.1a

$6x - 5y + z = d \rightarrow d = 6 \times 10 - 5 \times (-1) - 6 \Rightarrow 6x - 5y + z = 59$ $\frac{183}{\sqrt{6^2 + 5^2 + 1^2}} - \frac{59}{\sqrt{6^2 + 5^2 + 1^2}} = \dots$		
$\lambda = 2 \Rightarrow V = 9\sqrt{62} \times 2(6\mathbf{i} - 5\mathbf{j} + \mathbf{k}) = 9\sqrt{62} \times 2\sqrt{6^2 + 5^2 + 1} = \dots$ or $\lambda = 2 \Rightarrow V = (-54\mathbf{i} + 45\mathbf{j} - 9\mathbf{k}) \bullet 2(6\mathbf{i} - 5\mathbf{j} + \mathbf{k}) = \dots$	dM1	3.1a
= 1116	A1	1.1b
	(5)	
	(10 marks)	

Notes

M1: Recognises the requirement to find the equation of Π by forming the necessary scalar product

A1: Correct equation for Π

M1: Performs the key step of finding the value of the parameter that determines the required distance or vector that will allow the volume to be calculated

Alternatively find the equation of the plane containing P and then finds the shortest distance between the two planes

dM1: Completes the strategy by attempting the product of their answer to part (a) with the length of the relevant vector or attempts the magnitude of the scalar triple product using the relevant vectors

A1: Correct volume

(Corrected from the printed mark scheme: the second equation in the third row prints $\lambda = \dots (18)$; solving $54(10 + 6\lambda) - 45(-1 - 5\lambda) + 9(\lambda - 6) = 1647$ gives $\lambda = 2$.)

Alternative 1

Scheme	Marks	AO
$6x - 5y + z = d \rightarrow d = 6 \times 10 - 5 \times (-1) - 6$ $54x - 45y + 9z = d \rightarrow d = 54 \times 10 - 45 \times (-1) + 9(-6)$	M1	3.1a
$6x - 5y + z = 59$ o.e. $54x - 45y + 9z = 531$	A1	1.1b
$\frac{ (25 \times 6) + (-4 \times -5) + (13 \times 1) \pm 59 }{\sqrt{6^2 + 5^2 + 1^2}} = \dots \left\{ \frac{124}{\sqrt{62}} \right\}$ $\frac{ (25 \times 54) + (-4 \times -45) + (13 \times 9) \pm 531 }{\sqrt{54^2 + 45^2 + 9^2}} = \dots \left\{ \frac{124}{\sqrt{62}} \right\}$	M1	3.1a

$V = 9\sqrt{62} \times \frac{124}{\sqrt{62}}$	dM1	3.1a
$= 1116$	A1	1.1b
	(5)	

M1: Recognises the requirement to find the equation of the plane containing the parallelogram P by forming the necessary scalar product

A1: Correct equation for the plane

M1: Finds the shortest distance between the point $(25, -4, 13)$ and the plane

dM1: Dependent on previous method. Finds the required volume by multiplying their answer to (a) multiplied by the shortest distance

A1: Correct volume

(Corrected from the printed mark scheme: the plane is printed as $54x - 45y + 9z = -531$; the working above it gives $d = 54 \times 10 - 45 \times (-1) + 9(-6) = 531$.)

Alternative 2

Scheme	Marks	A0
$\begin{pmatrix} 25 \\ -4 \\ 13 \end{pmatrix} - \begin{pmatrix} 10 \\ -1 \\ -6 \end{pmatrix} = \begin{pmatrix} 15 \\ -3 \\ 19 \end{pmatrix}$	M1 A1	3.1a 1.1b
$\frac{\left \begin{pmatrix} 15 \\ -3 \\ 19 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix} \right }{\sqrt{6^2 + 5^2 + 1^2}} = \dots \text{ or } \begin{vmatrix} 3 & 4 & 2 \\ -3 & -1 & 13 \\ 15 & -3 & 19 \end{vmatrix} \text{ or } \begin{pmatrix} 15 \\ -3 \\ 19 \end{pmatrix} \cdot \begin{pmatrix} -54 \\ 45 \\ -9 \end{pmatrix}$	M1	3.1a
$V = 9\sqrt{62} \times \frac{124}{\sqrt{62}}$ Or $\begin{vmatrix} 3 & 4 & 2 \\ -3 & -1 & 13 \\ 15 & -3 & 19 \end{vmatrix} = 3(-19 + 39) - 4(-57 - 195) + 2(9 + 15)$ Or $\begin{pmatrix} 15 \\ -3 \\ 19 \end{pmatrix} \cdot \begin{pmatrix} -54 \\ 45 \\ -9 \end{pmatrix} = 15 \times -54 - 3 \times 45 + 19 \times -9$	dM1	3.1a
$= 1116$	A1	1.1b

M1: Find the vector between E and the position vector on Π

A1: Correct vector

M1: Finds the shortest distance between the planes or states determinant approach of 3 appropriate vectors for example $\overrightarrow{EF}$, $\overrightarrow{EH}$, $\overrightarrow{EA}$ where A is $(25, -4, 13)$ or the triple scalar product

dM1: Finds the required volume by multiplying their answer to (a) by the shortest distance or uses the determinant approach of 3 appropriate vectors or attempts the triple scalar product.

If no working is seen the answer must be correct for their vectors.

Using $\frac{1}{6}$ volume is dM0

A1: Correct volume