

QQQ – Further Pure 1 – Chapter 1 – Vectors (Version 1)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. The position vectors of the points A , B and C relative to an origin O are $\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, $7\mathbf{i} - 3\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j}$ respectively.

Find

- (a) $\overrightarrow{AC} \times \overrightarrow{BC}$, (4)
- (b) the area of triangle ABC , (2)
- (c) an equation of the plane ABC in the form $\mathbf{r} \cdot \mathbf{n} = p$ (2)

(Total 8 marks)

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2. With respect to a fixed origin O , the line l has equation

$$(\mathbf{r} - (12\mathbf{i} + 16\mathbf{j} - 8\mathbf{k})) \times (9\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}) = \mathbf{0}$$

The point A lies on l such that the direction cosines of $\overrightarrow{OA}$ with respect to the $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ axes are $\frac{3}{7}$, β and γ .

Determine the coordinates of the point A .

(7)

(Total 7 marks)

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3.

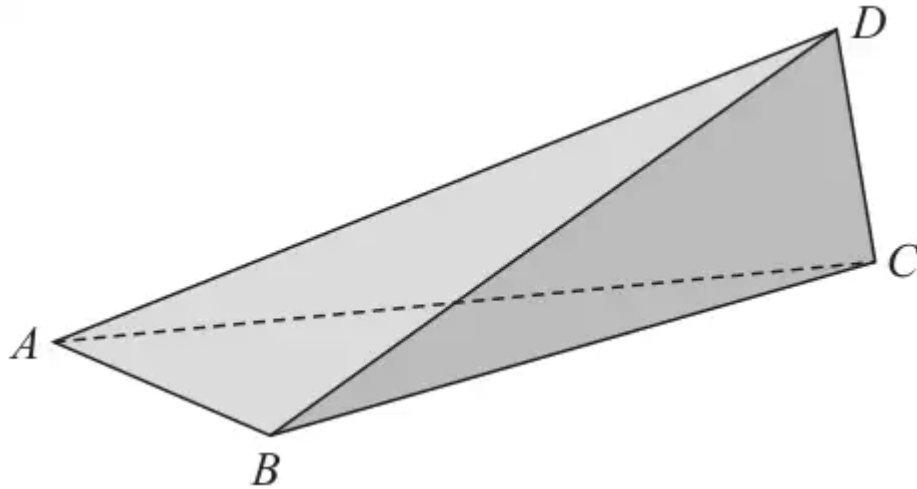


Figure 1

Figure 1 shows a sketch of a solid doorstop made of wood. The doorstop is modelled as a tetrahedron.

Relative to a fixed origin O , the vertices of the tetrahedron are $A (2, 1, 4)$, $B (6, 1, 2)$, $C (4, 10, 3)$ and $D (5, 8, d)$, where d is a positive constant and the units are in centimetres.

(a) Find the area of the triangle ABC . (4)

Given that the volume of the doorstop is 21 cm^3

(b) find the value of the constant d . (4)

(Total 8 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks
$\overrightarrow{AC} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}, \overrightarrow{BC} = -3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$	B1, B1
$\overrightarrow{AC} \times \overrightarrow{BC} = 10\mathbf{i} - 15\mathbf{j} + 30\mathbf{k}$	M1 A1
	(4)

Notes

a1B1: $\overrightarrow{AC} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ cao, any form

a2B1: $\overrightarrow{BC} = -3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ cao, any form

a1M1: Attempt to find cross product, modulus of one term correct.

a1A1: cao, any form.

Mark scheme (b)

Scheme	Marks
Area of triangle $ABC = \frac{1}{2} 10\mathbf{i} - 15\mathbf{j} + 30\mathbf{k} = \frac{1}{2} \sqrt{1225} = 17.5$	M1 A1
	(2)

Notes

b1M1: modulus of their answer to (a) – condone missing $\frac{1}{2}$ here. To finding area of triangle by correct method.

b1A1: cao.

Mark scheme (c)

Scheme	Marks
Equation of plane is $10x - 15y + 30z = -20$ or $2x - 3y + 6z = -4$	M1
So $\mathbf{r} \cdot (2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) = -4$ or correct multiple	A1
	(2)

(8 marks)

Notes

c1M1: [Using their answer to (a) to] find **equation** of plane. Look for $\mathbf{a} \cdot \mathbf{n}$ or $\mathbf{b} \cdot \mathbf{n}$ or $\mathbf{c} \cdot \mathbf{n}$ for p.

c1A1: cao

2.

Way 1

Scheme	Marks	AO
Position of A is given by $\overrightarrow{OA} = \begin{pmatrix} 12 + 9\lambda \\ 16 + 6\lambda \\ -8 + 2\lambda \end{pmatrix}$	B1	3.1a
So have $\frac{12 + 9\lambda}{\sqrt{(12 + 9\lambda)^2 + (16 + 6\lambda)^2 + (-8 + 2\lambda)^2}} = \frac{3}{7}$	M1	1.1b
$\Rightarrow 49(3(4 + 3\lambda))^2 =$ $9((12 + 9\lambda)^2 + (16 + 6\lambda)^2 + (-8 + 2\lambda)^2)$ $\Rightarrow 2880\lambda^2 + 7200\lambda + 2880 = 0$ or $2\lambda^2 + 5\lambda + 2 = 0$	M1 A1	3.1a 2.1
$\Rightarrow (2\lambda + 1)(\lambda + 2) = 0 \Rightarrow \lambda = \dots$	M1	1.1b
Substitutes a value of λ to find a position for A e.g. $\overrightarrow{OA} = \begin{pmatrix} 12 + 9\left(-\frac{1}{2}\right) \\ 16 + 6\left(-\frac{1}{2}\right) \\ -8 + 2\left(-\frac{1}{2}\right) \end{pmatrix} = \dots$	M1	1.1b
Coordinates of A are $\left(\frac{15}{2}, 13, -9\right)$ only	A1	2.3
	(7)	
	(7 marks)	

Notes

Way 1

B1: Starts a correct procedure by parametrising the line correctly.

M1: Uses the direction cosine of $\frac{3}{7}$ to form an equation in λ

M1: Realises need to square, to form quadratic in λ and gathers terms.

A1: Correct quadratic – three terms only or rearranged to complete square and solve, but need not have common factors all cancelled.

M1: Solves their three term quadratic, any valid method.

M1: Substitutes a value for λ into the equation of the line to find a position for A .

A1: Correct coordinates only

Way 2

Scheme	Marks	AO
Direction of $\overrightarrow{OA}$ is given by $\mathbf{d} = \begin{pmatrix} \frac{3}{7}k \\ \beta k \\ \gamma k \end{pmatrix}$ or use of $\left(\frac{3}{7}\right)^2 + \beta^2 + \gamma^2 = 1$	B1	3.1a
$\begin{pmatrix} \frac{3}{7}k - 12 \\ \beta k - 16 \\ \gamma k + 8 \end{pmatrix} \times \begin{pmatrix} 9 \\ 6 \\ 2 \end{pmatrix} = \mathbf{0} \Rightarrow \begin{cases} 6\left(\frac{3}{7}k - 12\right) - 9(\beta k - 16) = 0 \\ 2\left(\frac{3}{7}k - 12\right) - 9(\gamma k + 8) = 0 \\ 2(\beta k - 16) - 6(\gamma k + 8) = 0 \end{cases}$	M1	2.1
$\Rightarrow \beta k = \frac{2}{7}k + 8$ and $\gamma k = \frac{1}{3}\left(\frac{2}{7}k - 32\right)$ $\Rightarrow \frac{9k^2}{49} + \left(\frac{2}{7}k + 8\right)^2 + \frac{1}{9}\left(\frac{2}{7}k - 32\right)^2 = k^2 \Rightarrow 2k^2 - 7k - 490 = 0$	M1 A1	3.1a 1.1b
$\Rightarrow (2k - 35)(k + 14) = 0 \Rightarrow k = \dots$	M1	1.1b
$k > 0$ as direction cosine for first ordinate is positive, so need $k = \frac{35}{2}$ hence $\overrightarrow{OA} = \begin{pmatrix} \frac{3}{7} \times \frac{35}{2} \\ \frac{2}{7} \times \frac{35}{2} + 8 \\ \frac{1}{3}\left(\frac{2}{7} \times \frac{35}{2} - 32\right) \end{pmatrix} = \dots$	M1	2.3
Coordinates of A are $\left(\frac{15}{2}, 13, -9\right)$ only	A1	1.1b
	(7)	

B1: Starts correct procedure by using the direction cosines to parametrise $\overrightarrow{OA}$ or attempting to use the Pythagorean property of the direction cosines.

M1: Uses their $\overrightarrow{OA}$ as multiple of direction cosines in the line equation to produce simultaneous equations.

M1: Solves the system (no need to see check for consistency of third equation) to find βk and γk , or just β and γ in terms of k and proceeds to form a quadratic in k using the Pythagorean property of the direction

cosines.

A1: A correct quadratic in k reduced to three terms etc.

M1: Solves their three term quadratic, any valid method.

M1: Substitutes a value for λ into the equation of the line to find a position for A .

A1: Correct coordinates only

3.

Mark scheme (a)

Scheme	Marks	AO
$A(2, 1, 4), B(6, 1, 2), C(4, 10, 3), D(5, 8, d)$		
Way 1 Uses appropriate vectors in a correct method to make a complete attempt to find the area of triangle ABC .	M1	3.1b
$\vec{AB} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}, \vec{AC} = \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix}, \left\{ \vec{BC} = \begin{pmatrix} -2 \\ 9 \\ 1 \end{pmatrix} \right\}$	M1	1.1b
e.g. $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & -2 \\ 2 & 9 & -1 \end{vmatrix} = \dots$ or $\begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix} = \dots$	M1	1.1b
$= 18\mathbf{i} + 0\mathbf{j} + 36\mathbf{k}$		
$\text{Area } ABC = \frac{1}{2} \sqrt{(18)^2 + (0)^2 + (36)^2}$		
$\{ = 20.1246 \dots \} = 9\sqrt{5} \text{ (cm}^2\text{)} \text{ or awrt } 20.1 \text{ (cm}^2\text{)}$	A1	2.2a
	(4)	

Notes

Way 1

M1: Complete correct process of taking the vector product between 2 edges of triangle ABC , applying Pythagoras and multiplying the result by 0.5

M1: Uses a correct method to find any 2 edges of triangle ABC

M1: Attempts to take the vector cross product between 2 edges of triangle ABC

A1: Deduces the correct area of either $9\sqrt{5} \text{ (cm}^2\text{)}$ or awrt 20.1 (cm²)

Note: For Way 1 and Way 2, using any of $\overrightarrow{OA}$, $\overrightarrow{OB}$ or $\overrightarrow{OC}$ in their vector product is M0 M0 A0 A0

(a) Way 2

Scheme	Marks	A0
Uses appropriate vectors to find an angle or perpendicular height in triangle ABC and uses a correct method to make a complete attempt to find the area of triangle ABC .	M1	3.1b
$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix}, \left\{ \overrightarrow{BC} = \begin{pmatrix} -2 \\ 9 \\ 1 \end{pmatrix} \right\}$	M1	1.1b
Uses a correct method to find an angle or perpendicular height in triangle ABC	M1	1.1b
Note: $\hat{BAC} = 76.047 \dots, \hat{ABC} = 76.047 \dots, \hat{BCA} = 27.905 \dots$ or perpendicular height = 9		
Area $ABC = \frac{1}{2} \sqrt{86} \sqrt{20} \sin 76.047 \dots$ or $\frac{1}{2} \sqrt{86} \sqrt{86} \sin 27.905 \dots$ or $\frac{1}{2} \sqrt{20}(9)$		
$\{ = 20.1246 \dots \} = 9\sqrt{5} \text{ (cm}^2\text{)} \text{ or awrt } 20.1 \text{ (cm}^2\text{)}$	A1	2.2a
	(4)	

M1: See scheme

M1: Uses a correct method to find any 2 edges of triangle ABC

M1: Either

- finds an angle in ABC by using a correct scalar product method
- finds an angle in ABC by using the cosine rule in the correct direction
- realises triangle ABC is isosceles and applies Pythagoras in the correct direction to find the perpendicular height

A1: Deduces the correct area as either $9\sqrt{5} \text{ (cm}^2\text{)}$ or awrt $20.1 \text{ (cm}^2\text{)}$

(Corrected from the printed mark scheme: the Note in Way 2 is printed as

$\hat{BAC} = 27.905 \dots, \hat{ABC} = 76.047 \dots, \hat{BCA} = 76.047 \dots$; since $AC = BC = \sqrt{86}$, the angles at A and B are equal, and the angle at C is $27.905 \dots$)

(a) Way 3

Scheme	Marks	A0
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Complete attempt to find the area of triangle ABC by applying $\frac{1}{2} \overrightarrow{OA} \times \overrightarrow{OB} + \overrightarrow{OB} \times \overrightarrow{OC} + \overrightarrow{OC} \times \overrightarrow{OA} $ or equivalent	M1	3.1b
$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 4 \\ 6 & 1 & 2 \end{vmatrix} = \dots \text{ and } \overrightarrow{OB} \times \overrightarrow{OC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 1 & 2 \\ 4 & 10 & 3 \end{vmatrix} = \dots,$ $\text{and } \overrightarrow{OC} \times \overrightarrow{OA} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 10 & 3 \\ 2 & 1 & 4 \end{vmatrix} = \dots$	M1	1.1b
$\{\overrightarrow{OA} \times \overrightarrow{OB} + \overrightarrow{OB} \times \overrightarrow{OC} + \overrightarrow{OC} \times \overrightarrow{OA}\} = \begin{pmatrix} -2 \\ 20 \\ -4 \end{pmatrix} + \begin{pmatrix} -17 \\ -10 \\ 56 \end{pmatrix} + \begin{pmatrix} 37 \\ -10 \\ -16 \end{pmatrix}$	M1	1.1b
Area $ABC = \frac{1}{2} \sqrt{(18)^2 + (0)^2 + (36)^2}$		
$\{= 20.1246 \dots\} = 9\sqrt{5} \text{ (cm}^2\text{)} \text{ or awrt } 20.1 \text{ (cm}^2\text{)}$	A1	2.2a
	(4)	

M1: See scheme

M1: Attempts to apply $\overrightarrow{OA} \times \overrightarrow{OB}$, $\overrightarrow{OB} \times \overrightarrow{OC}$ and $\overrightarrow{OC} \times \overrightarrow{OA}$

A1: Attempts to add (as vectors) the results of applying $\overrightarrow{OA} \times \overrightarrow{OB}$, $\overrightarrow{OB} \times \overrightarrow{OC}$ and $\overrightarrow{OC} \times \overrightarrow{OA}$

A1: Deduces the correct area as either $9\sqrt{5} \text{ (cm}^2\text{)}$ or awrt $20.1 \text{ (cm}^2\text{)}$

Mark scheme (b)

Scheme	Marks	AO
Finds appropriate vectors to form the equation volume tetrahedron $ABCD = 21$ to give a linear equation in d Note: The volume must include $\frac{1}{6}$	M1	3.1a
e.g. $\left \begin{pmatrix} 3 \\ 7 \\ d-4 \end{pmatrix} \cdot \begin{pmatrix} 18 \\ 0 \\ 36 \end{pmatrix} \right = \dots \text{ or } \begin{vmatrix} 4 & 0 & -2 \\ 2 & 9 & -1 \\ 3 & 7 & d-4 \end{vmatrix} = \dots$	M1	1.1b
$= 54 + 36d - 144 $ or $ 4(9d - 36 + 7) - 2(14 - 27) \{= 36d - 90 \}$	A1	1.1b

$\left\{ \frac{1}{6} 36d - 90 = 21 \Rightarrow 36d - 90 = 126 \Rightarrow \right\} d = 6$	A1	1.1b
	(4)	
	(8 marks)	

Notes

M1: See scheme

M1: Uses appropriate vectors in an attempt at the scalar triple product

A1: Correct applied expression for the scalar triple product (allow $\pm$ and ignore modulus sign)

A1: Correct solution leading to $d = 6$

Note: Using any of $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ or $\overrightarrow{OD}$ in their scalar triple product is M0 M0 A0 A0

Note: Some vector product calculations for reference:

$$|\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})| = \begin{vmatrix} 3 & 7 & d-4 \\ 4 & 0 & -2 \\ 2 & 9 & -1 \end{vmatrix} = \left| \begin{pmatrix} 3 \\ 7 \\ d-4 \end{pmatrix} \cdot \begin{pmatrix} 18 \\ 0 \\ 36 \end{pmatrix} \right| = |54 + 36d - 144| = |36d - 90|$$

$$|\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD})| = \begin{vmatrix} 4 & 0 & -2 \\ 2 & 9 & -1 \\ 3 & 7 & d-4 \end{vmatrix} = \left| \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 9d-29 \\ 5-2d \\ -13 \end{pmatrix} \right| = |36d - 116 + 26| = |36d - 90|$$

$$|\overrightarrow{AC} \cdot (\overrightarrow{AB} \times \overrightarrow{AD})| = \begin{vmatrix} 2 & 9 & -1 \\ 4 & 0 & -2 \\ 3 & 7 & d-4 \end{vmatrix} = \left| \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 14 \\ 10-4d \\ 28 \end{pmatrix} \right| = |28 + 90 - 36d - 28| = |90 -$$