

QQQ – Further Mechanics 2 – Chapter 5 – Dynamics (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. A spacecraft S of mass m moves in a straight line towards the centre of the earth. The earth is modelled as a fixed sphere of radius R . When S is at a distance x from the centre of the earth, the force exerted by the earth on S is directed towards the centre of the earth and has magnitude $\frac{k}{x^2}$, where k is a constant.

(a) Show that $k = mgR^2$. (2)

Given that S starts from rest when its distance from the centre of the earth is $2R$, and that air resistance can be ignored,

(b) find the speed of S as it crashes into the surface of the earth. (7)

(Total 9 marks)

Edexcel M3 June 2007 Q3

2. Throughout this question, use $g = 10 \text{ m s}^{-2}$

A light elastic string has natural length 1.25 m and modulus of elasticity 25 N.

A particle P of mass 0.5 kg is attached to one end of the string. The other end of the string is attached to a fixed point A . Particle P hangs freely in equilibrium with P vertically below A

The particle is then pulled vertically down to a point B and released from rest.

(a) Show that, while the string is taut, P moves with simple harmonic motion with period $\frac{\pi}{\sqrt{10}}$ seconds. (6)

The maximum kinetic energy of P during the subsequent motion is 2.5 J.

(b) Show that $AB = 2 \text{ m}$ (3)

The particle returns to B for the first time T seconds after it was released from rest at B

(c) Find the value of T (5)

(Total 14 marks)

Edexcel A2 June 2022 Q8

Solutions

1.

Mark scheme (a)

Scheme	Marks
At surface $\frac{k}{R^2} = mg \Rightarrow k = mgR^2$ * cso	M1 A1
	(2)

Mark scheme (b)

Scheme	Marks
N2L $m\ddot{x} = -\frac{mgR^2}{x^2}$	
$v \frac{dv}{dx} = -\frac{gR^2}{x^2}$ or $\frac{d}{dx} \left(\frac{1}{2}v^2 \right) = -\frac{gR^2}{x^2}$	M1
$\int v dv = -gR^2 \int \frac{1}{x^2} dx$ or $\frac{1}{2}v^2 = -gR^2 \int \frac{1}{x^2} dx$	M1
$\frac{1}{2}v^2 = \frac{gR^2}{x} (+C)$	A1
$x = 2R, v = 0 \Rightarrow C = -\frac{gR}{2}$	M1 A1
$v^2 = \frac{2gR^2}{x} - gR$	
At $x = R, v^2 = \frac{2gR^2}{R} - gR$	M1
$v = \sqrt{(gR)}$	A1
	(7)
	(9 marks)

2.

Mark scheme (a)

Scheme	Marks	A0
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At equilibrium: $0.5g = \frac{25e}{1.25}$, $e = \frac{0.5 \times 10 \times 1.25}{25} = \frac{1}{4}$	B1	3.3
For taut string, when distance x from equilibrium, equation of motion	M1	2.1
Alternative for M1: Conservation of energy using a known point (E or B) and a general position: From E : $\frac{25e^2}{2 \times 1.25} + KE(\text{constant} \neq 0) + 0.5gx = \frac{25(e+x)^2}{2 \times 1.25} + \frac{1}{2}0.5v^2 + 0(\text{GPE})$ and differentiate wrt x for M1 $\Rightarrow 0.5g = \frac{25(e+x)}{1.25} + \frac{1}{2}v \frac{dv}{dx}$		
$\frac{25(e+x)}{1.25} - 0.5g = -0.5\ddot{x}$	A1ft	1.1b
$\ddot{x} = -40x$ hence SHM*	A1*	2.2a
Periodic time:	M1	3.4
$T = \frac{2\pi}{\sqrt{40}} = \frac{\pi}{\sqrt{10}} *$	A1*	2.2a
	(6)	

Notes

B1: Correct only Award if see $mg = \frac{\lambda e}{l}$ used in their equation of motion

M1: Equation of motion with x measured from the equilibrium position. Need all terms and dimensionally correct. Allow with their $e \neq 0$. Condone sign errors. Allow with a in place of $\ddot{x}$

A1ft: Correct unsimplified equation with their e or $e \neq 0$.

Could have the negative of the whole equation
or x replaced with $-x$ throughout

A1*: Reach **given conclusion** from correct working.
Condone correct conclusion without explanation

M1: Use the model to find the periodic time: $T = \frac{2\pi}{\omega}$
From an equation of the form $\ddot{x} = -\omega^2 x$

A1*: Obtain given answer from correct **working** throughout.

Available if only error is not to conclude SHM

Mark scheme (b)

Scheme	Marks	AO
Max KE = 2.5 = $\frac{1}{2} \times \frac{1}{2} \times \max v^2 \Rightarrow \max v^2 = 10$	B1	1.2
Max speed = $a\omega$: $\sqrt{10} = a\sqrt{40}$	M1	3.4
$AB = 1.25 + \frac{1}{4} + \frac{1}{2} = 2 \text{ (m)}$ *	A1*	1.1b
	(3)	

Notes

B1: Use the KE to find $\max v$ or $\max v^2$

M1: Use the model to find the amplitude of the motion

A1*: Obtain the given answer from correct working.

Alternative (b)

Scheme	Marks	AO
Energy: $\frac{25e^2}{2.5} + 2.5 + 0.5ga = \frac{25(e+a)^2}{2.5}$	B1	
Solve for a	M1	
$AB = 1.25 + \frac{1}{4} + \frac{1}{2} = 2 \text{ (m)}$ *	A1*	1.1b
	(3)	

B1: Using correct λ and l and e or their e

M1: Requires an energy equation with all the right terms

A1*: Obtain the given answer from correct working.

Mark scheme (c)

Scheme	Marks	AO
$a = 0.5, x = 0.5 \cos \sqrt{40}t$	B1	2.2a
$-0.25 = 0.5 \cos \sqrt{40}t \Rightarrow t = 0.3311 \dots$	M1	3.1a

$v^2 = 40 (0.5^2 - 0.25^2) = \frac{15}{2}$	M1	3.4
Total time = $2 \times 0.3311 \dots + \frac{2 \times \sqrt{7.5}}{10}$	DM1	3.1a
= 1.2 (s) or better	A1	2.2a
	(5)	
	(14 marks)	

Notes

B1: Correct equation for SHM seen or implied

M1: Find the time until the string goes slack

If working from $x = 0.5 \sin \sqrt{40}t$ need $\frac{T}{4} + \frac{1}{\sqrt{40}} \sin^{-1} \frac{1}{2}$

M1: Use the model to find v or v^2 at the instant the string goes slack ($v = 2.738 \dots$)

Using SHM formula or conservation of energy.

M1: Complete method to find the total time until return to B

Requires the preceding M marks

If they use suvat to find the time as a projectile it must be a complete method e.g. $\sqrt{\frac{15}{2}} = -\sqrt{\frac{15}{2}} + gt$

or a combination of $v^2 = u^2 + 2as$ and $s = ut + \frac{1}{2}at^2$

A1: = 1.2 (s) or better Condone an answer to > 2 s.f.

Not scored if they have used 9.8.