

QQQ – Further Mechanics 2 – Chapter 5 – Dynamics (Version 1)

Total Marks: 26

(26 = Platinum, 23 = Gold, 21 = Silver, 18 = Bronze)

1. A particle P of mass 0.5 kg moves on the positive x -axis under the action of a single force directed towards the origin O . At time t seconds the distance of P from O is x metres, the magnitude of the force is $0.375x^2$ N and the speed of P is v m s⁻¹.

When $t = 0$, $OP = 8$ m and P is moving towards O with speed 2 m s⁻¹.

(a) Show that $v^2 = 260 - \frac{1}{2}x^3$. (4)

(b) Find the distance of P from O at the instant when $v = 5$. (2)

(Total 6 marks)

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2. A particle P of mass m moves in a straight line away from the centre of the Earth. The Earth is modelled as a sphere of radius R . When P is at a distance x , $x \geq R$, from the centre of the Earth, the force exerted by the Earth on P is directed towards the centre of the Earth and has magnitude $\frac{mgR^2}{x^2}$. When P is at a distance $2R$ from the surface of the Earth, the speed of P is $\sqrt{\frac{gR}{3}}$.

Assuming that air resistance can be ignored, find the distance of P from the surface of the Earth

when the speed of P is $2\sqrt{\frac{gR}{3}}$. (7)

(Total 7 marks)

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3. The points A and B lie on a smooth horizontal surface with $AB = 4.5$ m.

A light elastic string has natural length 1.5 m and modulus of elasticity 15 N. One end of the string is attached to A and the other end of the string is attached to B . A particle, P , of mass 0.2 kg, is attached to the stretched string so that APB is a straight line and $AP = 1.5$ m. The particle rests in equilibrium on the surface.

The particle is now moved directly towards A and is held on the surface so APB is a straight line with $AP = 1$ m.

The particle is released from rest.

(a) Prove that P moves with simple harmonic motion. **(5)**

(b) Find

(i) the maximum speed of P during the motion,

(ii) the maximum acceleration of P during the motion. **(3)**

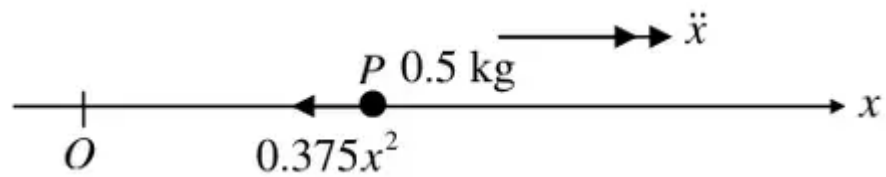
(c) Find the total time, in each complete oscillation of P , for which the speed of P is greater than 5 m s^{-1} . **(5)**

(Total 13 marks)

Solutions

1.

Mark scheme (a)



Scheme	Marks
$0.5v \frac{dv}{dx} = -0.375x^2$	M1
$\frac{1}{2}v^2 = -0.25x^3 + c$	M1 A1
$t = 0, v = 2, x = 8$ $\frac{1}{2} \times 2^2 = -0.25 \times 8^3 + c$ $c = 130$	
$\therefore v^2 = -\frac{1}{2}x^3 + 260$ *	A1
	(4)

Mark scheme (b)

Scheme	Marks
$v = 5 \quad x^3 = 520 - 50$	M1
$x = 7.77$	A1
	(2)
	(6 marks)

2.

Scheme	Marks
$mv \frac{dv}{dx} = -\frac{mgR^2}{x^2}$	M1

$\frac{1}{2}v^2 = - \int gR^2 x^{-2} dx$	
$\frac{1}{2}v^2 = gR^2 x^{-1} (+c)$	dM1A1
$x = 3R \quad v = \sqrt{\frac{gR}{3}} \Rightarrow c = \frac{1}{2} \frac{gR}{3} - \frac{gR^2}{3R} = -\frac{gR}{6}$	dM1
$v = 2\sqrt{\frac{gR}{3}} \quad 2\frac{gR}{3} = \frac{gR^2}{x} - \frac{gR}{6} \quad x = \dots$	dM1
$x = \frac{6R}{5}$	A1
Dist from surface = $\frac{6R}{5} - R = \frac{R}{5}$ oe	Alcso
	(7 marks)

Notes

M1 Attempting an equation of motion with correct number of terms and acceleration $v \frac{dv}{dx}$.

Allow with minus missing. Can be given by implication if acceleration is integrated to $\frac{1}{2}v^2$

dM1 Attempting the integration of both sides of their equation. $x^{-2} \rightarrow x^{-1}$ Depends on the first M mark.

A1 Correct equation after correct integration. Constant of integration may be missing.

Double sign error scores A0 here.

dM1 Substitute $x = 3R \quad v = \sqrt{\frac{gR}{3}}$ and obtain an expression for c . Depends on the first M mark providing an attempt at integration is seen. (eg $x^{-2} \rightarrow x^{-3}$ could score M1M0A0dM1)

dM1 Substitute $v = 2\sqrt{\frac{gR}{3}}$ in their expression for v^2 and solve for x Depends on the first M mark.

A1 Correct x Double sign error scores A0 here.

Alcso Correct answer from completely correct working.

ALT 1 Definite Integration:

$mv \frac{dv}{dx} = -\frac{mgR^2}{x^2}$	M1
$\int_{\sqrt{\frac{gR}{3}}}^{2\sqrt{\frac{gR}{3}}} v dv = - \int_{3R}^X gR^2 x^{-2} dx$	

$\left[\frac{1}{2}v^2\right]_{\sqrt{\frac{gR}{3}}}^{2\sqrt{\frac{gR}{3}}} = [gR^2x^{-1}]_{3R}^X$	dM1A1
$\frac{1}{2} \times 4 \frac{gR}{3} - \frac{1}{2} \frac{gR}{3} = \frac{gR^2}{X} - \frac{gR}{3}$	dM1
$\frac{2}{3} - \frac{1}{6} = \frac{R}{X} - \frac{1}{3} \quad \frac{R}{X} = \frac{5}{6}$	
$X = \frac{6}{5}R$	dM1A1
Dist from surface = $\frac{6R}{5} - R = \frac{R}{5}$ oe	A1 cso [7]

M1 Attempting an equation of motion with correct number of terms and acceleration $v \frac{dv}{dx}$.

Allow with minus missing.

dM1 Attempting the integration of both sides of their equation. $x^{-2} \rightarrow x^{-1}$ Depends on the first M mark. Limits not needed (ignore any shown)

A1 Correct integration. Ignore any limits shown.

dM1 Substitute correct limits. May be as shown or **both** sets reversed. Depends on the first M mark.

dM1 Solve to $X = \dots$ Depends on the first M mark.

A1 Correct X

A1cso Correct answer from completely correct working.

ALT 2 Energy:

Variable force, so no integration implies no marks

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \int F dx = \int -\frac{mgR^2}{x^2} dx$	M1 (minus and limits may be missing)
$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \left[\frac{mgR^2}{x}\right]_{3R}^X$	dM1A1 Integrate RHS (as above) Inconsistent signs scores dM1A0)
$\frac{1}{2}m \times \frac{4gR}{3} - \frac{1}{2}m \times \frac{gR}{3} = \frac{mgR^2}{X} - \frac{mgR}{3}$	dM1 Sub correct limits
$X = \frac{6}{5}R,$ Dist from surface = $\frac{6R}{5} - R = \frac{R}{5}$ oe	dM1A1, A1cso As alt 1

Mark scheme (a)

Scheme	Marks	AO
Use of Hooke's law seen	B1	2.1
Let x be the displacement from equilibrium, then $T_B - T_A = -0.2\ddot{x}$.	M1	3.1a
$\frac{15(2+x)}{1} - \frac{15(1-x)}{0.5} = -0.2\ddot{x}$	A1 A1	1.1b 1.1b
$\Rightarrow \ddot{x} = -225x = -15^2x$, which is of the form $\ddot{x} = -\omega^2x$, so SHM *	A1*	3.2a
	(5)	

Notes

B1: Use of Hooke's law seen at least once (with natural length ≤ 1.5).

M1: Form equation of motion involving a difference of 2 tensions. Must be dimensionally correct. Need all terms. Condone incorrect lengths. Condone sign errors.

A1: Unsimplified equation with at most one error

A1: Correct unsimplified equation

A1*: All correct and justification of SHM – comment required

Mark scheme (b)

Scheme	Marks	AO
$a = 0.5$, $\omega = 15$ or their ω	B1ft	1.1b
Max speed = $a\omega = 7.5 \text{ (m s}^{-1}\text{)}$	B1ft	1.2
Max acceleration = $a\omega^2 = 112.5 \text{ (m s}^{-2}\text{)}$	B1ft	1.2
	(3)	

Notes

B1ft: Correct values seen or implied. a must be correct but follow their ω from (a)

B1ft: Follow their a, ω

B1ft: Follow their a, ω

Mark scheme (c)

Scheme	Marks	AO
$x = 0.5 \cos 15t \Rightarrow \dot{x} = -7.5 \sin 15t$	B1ft	2.2a
$ \dot{x} = 7.5 \sin 15t = 5 \Rightarrow \sin 15t = \frac{2}{3} \Rightarrow t = \dots$	M1	1.1b
$t = 0.0486 \dots$ (s)	A1	1.1b
Complete strategy to find the required time	M1	3.1a
Speed > 5 for $\frac{2\pi}{15} - 4t = 0.22$ (s)	A1	1.1b
	(5)	
	(13 marks)	

Notes

B1ft: Use correct equation for speed ($-a\omega \sin \omega t$)

Follow their a and ω . Allow sin or cos form

Can also use correct expression for x combined with use of $v^2 = \omega^2 (a^2 - x^2)$

M1: Find a relevant time e.g. time taken for 0 m s^{-1} to 5 m s^{-1}

A1: Seen or implied

M1: Complete strategy for the required time – e.g. find value for t when $v = 5$ and use symmetry and periodic time

A1: 2 s.f. or better (0.22428...)