

QQQ – Further Mechanics 2 – Chapter 4 – Kinematics (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. A particle P moves on the positive x -axis. When the distance of P from the origin O is x metres, the acceleration of P is $(7 - 2x) \text{ m s}^{-2}$, measured in the positive x -direction. When $t = 0$, P is at O and is moving in the positive x -direction with speed 6 m s^{-1} . Find the distance of P from O when P first comes to instantaneous rest. **(6)**

(Total 6 marks)

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2. A particle P is moving along the x -axis.

At time t seconds, $t \geq 0$, P has acceleration $a \text{ m s}^{-2}$ and velocity $v \text{ m s}^{-1}$ in the direction of x increasing, where

$$v = e^{2t} + 6e^t - kt$$

and k is a positive constant.

When $t = \ln 2$, $a = 0$

- (a) Find the value of k . **(4)**

When $t = 0$, the particle passes through the fixed point A .

When $t = \ln 2$, the particle is d metres from A .

- (b) Showing all stages of your working, find the value of d correct to 2 significant figures.

[Solutions relying entirely on calculator technology are not acceptable.] **(4)**

(Total 8 marks)

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3. A particle P is moving along the x -axis. At time t seconds, P has velocity $v \text{ m s}^{-1}$ in the positive x direction and acceleration $a \text{ m s}^{-2}$ in the positive x direction.

In a model of the motion of P

$$a = 4 - 3v$$

When $t = 0$, $v = 0$

(a) Use integration to show that $v = k(1 - e^{-3t})$, where k is a constant to be found. **(7)**

When $t = 0$, P is at the origin O

(b) Find, in terms of t only, the distance of P from O at time t seconds. **(4)**

(Total 11 marks)

Solutions

1.

Scheme	Marks
$v \frac{dv}{dx} = 7 - 2x$	M1
$\frac{1}{2}v^2 = 7x - x^2 \quad (+c)$	M1A1
$x = 0 \quad v = 6 \Rightarrow c = 18$	A1
$v = 0 \quad x^2 - 7x - 18 = 0$ $(x + 2)(x - 9) = 0$	M1
$\therefore x = 9$	A1
	(6 marks)

2.

Mark scheme (a)

Scheme	Marks	AO
Use of $a = \frac{dv}{dt}$	M1	3.1a
$a = 2e^{2t} + 6e^t - k$	A1	1.1b
Substitute $t = \ln 2$ into their acceleration and solve for k	M1	1.1b
$k = 20$	A1	2.2a
	(4)	

Notes

M1: Use the model and differentiate v to obtain a .

Obtain form $pe^{2t} + qe^t - k$

A1: Correct only

M1: $(2 \times 4 + 6 \times 2 - k = 0)$ Their acceleration must come from an attempt to differentiate.

A1: Correct only

Mark scheme (b)

Scheme	Marks	AO
Use of $v = \frac{dx}{dt}$	M1	2.1
$x = \frac{1}{2}e^{2t} + 6e^t - \frac{k}{2}t^2 (+C)$	A1ft	1.1b
Correct use of boundary conditions in their x	M1	3.1a
$x = 2 + 12 - 10(\ln 2)^2 - \left(\frac{1}{2} + 6\right) (= 7.5 - 10(\ln 2)^2) = 2.7$ to 2 s.f.	A1	1.1b
	(4)	
	(8 marks)	

Notes

M1: Integrate v to obtain x . Obtain form $ae^{2t} + be^t + ct^2$

A1ft: Allow in k or their k

M1: Use the model to evaluate constant of integration or use boundary conditions as limits in a definite integral. Their x must come from an attempt to integrate.

A1: 2sf only

3.

Mark scheme (a)

Scheme	Marks	AO
$a = 4 - 3v \Rightarrow \int \frac{1}{4 - 3v} dv = \int 1 dt$	M1	2.1
Integrate both sides of the equation	M1	1.1b
$\Rightarrow -\frac{1}{3} \ln 4 - 3v = t (+C)$	A1	1.1b
Use $t = 0, v = 0$	M1	3.4
$t = \frac{1}{3} \ln \left(\frac{4}{4 - 3v} \right)$	A1	1.1b
$\Rightarrow e^{3t} = \frac{4}{4 - 3v}, \quad 4 - 3v = 4e^{-3t}$	M1	1.1b

$v = \frac{4}{3} (1 - e^{-3t}) *$	A1*	2.2a
	(7)	

Notes

M1: Use $a = \frac{dv}{dt}$ and separate the variables to form integrals in v and t

M1: Integrate to obtain terms $p \ln(4 - 3v)$ and qt

A1: Or equivalent. Accept with brackets in place of modulus signs. Condone missing constant of integration

M1: Use boundary conditions in the model to evaluate constant of integration or as limits on a definite integral

A1: Or equivalent

M1: Rearrange to obtain expression for v in terms of t

A1*: Obtain **given form** with $k = \frac{4}{3}$ from correct working

Mark scheme (b)

Scheme	Marks	AO
$\frac{dx}{dt} = k(1 - e^{-3t}) \Rightarrow \int 1 dx = \int k(1 - e^{-3t}) dt$	M1	3.3
$x = k \left(t + \frac{1}{3}e^{-3t} \right) (+C) = \frac{4}{3} \left(t + \frac{1}{3}e^{-3t} \right) (+C)$	M1	1.1b
Use $t = 0, x = 0$	M1	3.4
$x = k \left(t + \frac{1}{3}e^{-3t} - \frac{1}{3} \right) = \frac{4}{3} \left(t + \frac{1}{3}e^{-3t} - \frac{1}{3} \right)$	A1ft	1.1b
	(4)	
	(11 marks)	

Notes

M1: Use $v = \frac{dx}{dt}$ to form integrals in x and t

M1: Integrate to obtain $\lambda t + \mu e^{-3t} (+C)$

M1: Use boundary conditions in the model to evaluate constant of integration or as limits on a definite integral

A1ft: Any equivalent form. Follow their k