

QQQ – Further Mechanics 2 – Chapter 3 – Further Centres of Mass (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1.

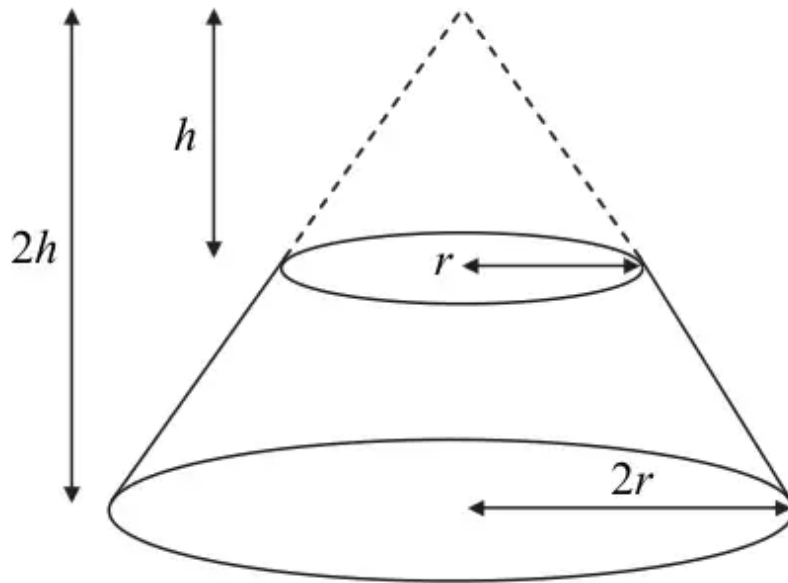


Figure 1

A uniform solid S is formed by taking a uniform solid right circular cone, of base radius $2r$ and height $2h$, and removing the cone, with base radius r and height h , which has the same vertex as the original cone, as shown in Figure 1.

(a) Show that the distance of the centre of mass of S from its larger plane face is $\frac{11}{28}h$. (5)

The solid S lies with its larger plane face on a rough table which is inclined at an angle θ° to the horizontal. The table is sufficiently rough to prevent S from slipping. Given that $h = 2r$,

(b) find the greatest value of θ for which S does not topple. (3)

(Total 8 marks)

2.

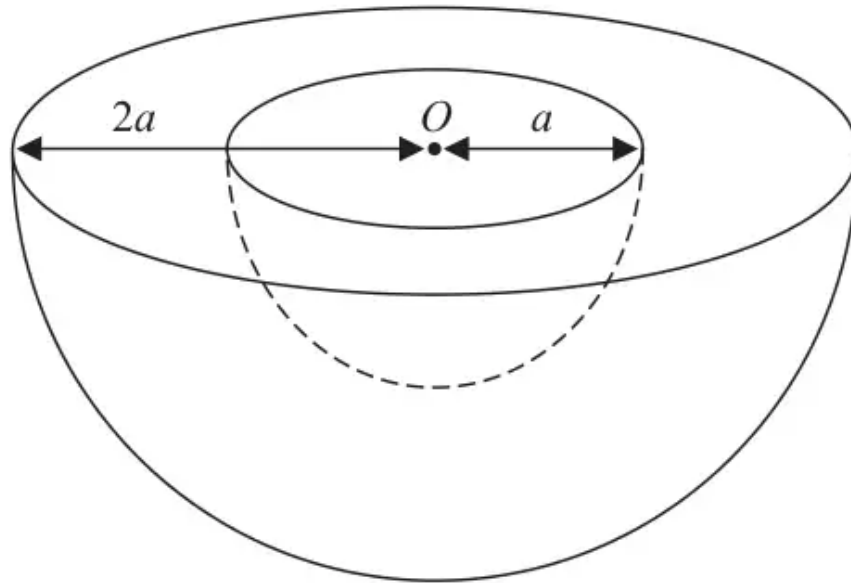


Figure 2

A uniform solid hemisphere H has radius $2a$. A solid hemisphere of radius a is removed from the hemisphere H to form a bowl. The plane faces of the hemispheres coincide and the centres of the two hemispheres coincide at the point O , as shown in Figure 2.

The centre of mass of the bowl is at the point G .

(a) Show that $OG = \frac{45a}{56}$ (4)

Figure 3 below shows a cross-section of the bowl which is resting in equilibrium with a point P on its curved surface in contact with a rough plane. The plane is inclined to the horizontal at an angle α and is sufficiently rough to prevent the bowl from slipping. The line OG is horizontal and the points O , G and P lie in a vertical plane which passes through a line of greatest slope of the inclined plane.

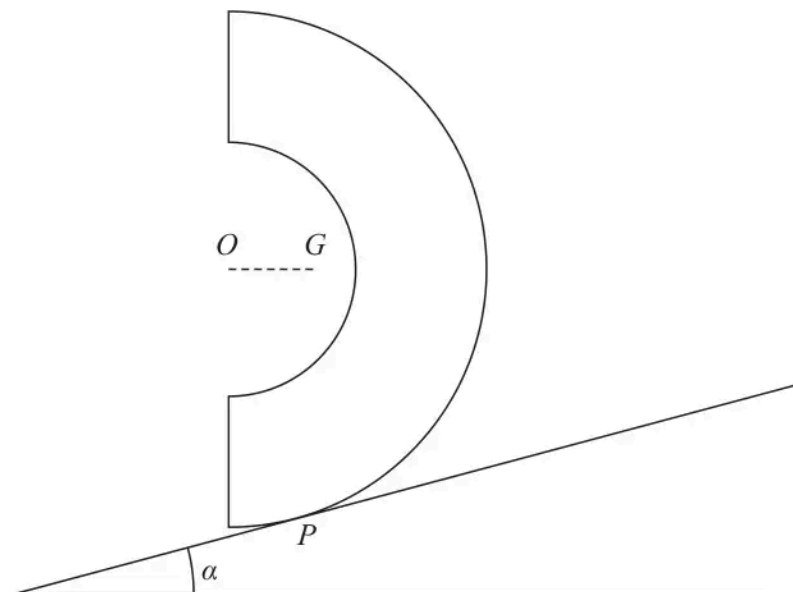


Figure 3

(b) Find the size of α , giving your answer in degrees to 3 significant figures.

(2)

(Total 6 marks)

3. [In this question, you may assume that the centre of mass of a circular arc, radius r , with angle at centre 2α , is a distance $\frac{r \sin \alpha}{\alpha}$ from the centre.]

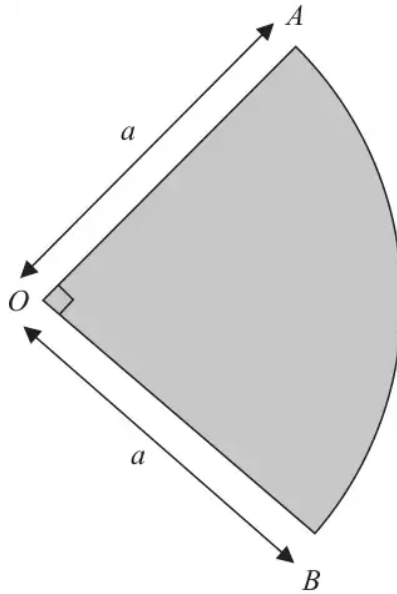


Figure 5

A thin non-uniform metal plate is in the shape of a sector OAB of a circle with centre O and radius a . The angle $AOB = \frac{\pi}{2}$, as shown in Figure 5.

The plate is modelled as a non-uniform lamina.

The mass per unit area of the lamina, at any point P of the lamina, is modelled as $k(OP)^2$,

where $k = \frac{4\lambda}{\pi a^4}$ and λ is a constant.

Using the model,

(a) find the mass of the plate in terms of λ , (5)

(b) find, in terms of a , the distance of the centre of mass of the plate from O . (4)

(Total 9 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks												
<table><tr><td></td><td>Large cone</td><td>small cone</td><td>S</td></tr><tr><td>Vol.</td><td>$\frac{1}{3}\pi(2r)^2(2h)$</td><td>$\frac{1}{3}\pi r^2 h$</td><td>$\frac{7}{3}\pi r^2 h$</td></tr><tr><td>C of M</td><td>$\frac{1}{2}h,$</td><td>$\frac{5}{4}h$</td><td>$\bar{x}$</td></tr></table> Vol.: (accept ratios 8 : 1 : 7) C of M: (or equivalent)		Large cone	small cone	S	Vol.	$\frac{1}{3}\pi(2r)^2(2h)$	$\frac{1}{3}\pi r^2 h$	$\frac{7}{3}\pi r^2 h$	C of M	$\frac{1}{2}h,$	$\frac{5}{4}h$	$\bar{x}$	B1 B1, B1
	Large cone	small cone	S										
Vol.	$\frac{1}{3}\pi(2r)^2(2h)$	$\frac{1}{3}\pi r^2 h$	$\frac{7}{3}\pi r^2 h$										
C of M	$\frac{1}{2}h,$	$\frac{5}{4}h$	$\bar{x}$										
$\frac{8}{3}\pi r^2 h.\frac{1}{2}h - \frac{1}{3}\pi r^2 h.\frac{5}{4}h = \frac{7}{3}\pi r^2 h.\bar{x}$ or equivalent	M1												
$\rightarrow \bar{x} = \frac{11}{28}h$ *	A1												
	(5)												

Notes

Centres of mass may be measured from another point (e.g. centre of small circle, or vertex). The Method mark will then require a complete method (Moments and subtraction) to give required value for $\bar{x}$. However B marks can be awarded for correct values if the candidate makes the working clear.

Mark scheme (b)

Scheme	Marks
$\tan \theta = \frac{2r}{\bar{x}} = \frac{2r}{\frac{11}{28}h}, = \frac{2r}{\frac{11}{14}r} = \frac{28}{11}$	M1, A1
$\theta \approx 68.6^\circ$ or 1.20 radians	A1
	(3)
	(8 marks)

Notes

(Special case – obtains complement by using $\tan \theta = \frac{\bar{x}}{2r}$ giving 21.4° or .374 radians M1A0A0)

(Corrected from the printed mark scheme: the special case is printed as $\tan \theta = \frac{2r}{\bar{x}}$.)

2.

Mark scheme (a)

Scheme	Marks	AO
Mass ratios: $\frac{2}{3}\pi(2a)^3, \frac{2}{3}\pi a^3, \frac{2}{3}\pi(2a)^3 - \frac{2}{3}\pi a^3$ (8, 1, 7)	B1	1.2
Distances: $\frac{3}{8}(2a), \frac{3}{8}a, \bar{x}$	B1	1.2
$\left(\frac{2}{3}\pi(2a)^3 - \frac{2}{3}\pi a^3\right) \bar{x} = \frac{2}{3}\pi(2a)^3 \times \frac{3}{8}(2a) - \frac{2}{3}\pi a^3 \times \frac{3}{8}a$	M1	3.1a
$\bar{x} = \frac{45a}{56}$ *	A1*	2.2a
	(4)	

Notes

B1: Correct unsimplified (8, 1, 7)

B1: Correct unsimplified but distances could be measured from a parallel axis

M1: All terms needed and must be dimensionally correct

A1*: Correct answer correctly derived

Mark scheme (b)

Scheme	Marks	AO
Use of appropriate trig ratio e.g. $\sin \alpha = \frac{\frac{45a}{56}}{2a}$	M1	3.1a
$\alpha = 23.7^\circ$ (3 sf)	A1	1.1b
	(2)	
	(6 marks)	

Notes

M1: Must be using an appropriate trig ratio

A1: Cao to 3SF

3.

Mark scheme (a)

Scheme	Marks	AO
Use of an appropriate element (quarter of a circle)	M1	2.1
$\delta A \simeq \frac{1}{2}\pi x \delta x$	A1	1.1b
$\delta m \simeq \frac{1}{2}\pi x \delta x \times \frac{4\lambda}{\pi a^4} x^2 \quad \left(= \frac{2\lambda}{a^4} x^3 \delta x \right)$	A1	3.4
$M = \int_0^a \frac{2\lambda}{a^4} x^3 dx$	M1	2.1
$M = \frac{1}{2}\lambda$	A1	1.1b
	(5)	

Notes

M1: Use of an appropriate element (may be implied)

A1: Correct expression for area of element (may be implied)

A1: Use of proportionality model to obtain mass of element (may be implied)

M1: Integrating with correct limits

A1: cao

Mark scheme (b)

Scheme	Marks	AO
Use of " $\bar{x} = \frac{1}{M} \int x dm$ "	M1	3.4
$= \frac{1}{M} \int_0^a \left(\frac{2\sqrt{2}x}{\pi} \right) \frac{2\lambda}{a^4} x^3 dx$	A1	1.1b
Substitute for M , integrate and sub. in limits	M1	3.4
$\bar{x} = \frac{8\sqrt{2}a}{5\pi}$	A1	1.1b
	(4)	
	(9 marks)	

Notes

M1: Use the model and correct method

A1: Correct integral

M1: Use the model to complete the equation

A1: cao