

QQQ – Further Mechanics 2 – Chapter 3 – Further Centres of Mass (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. A uniform solid is formed by rotating the region enclosed between the curve with equation $y = \sqrt{x}$, the x -axis and the line $x = 4$, through one complete revolution about the x -axis. Find the distance of the centre of mass of the solid from the origin O . **(5)**

(Total 5 marks)

Edexcel M3 June 2006 Q1

2.

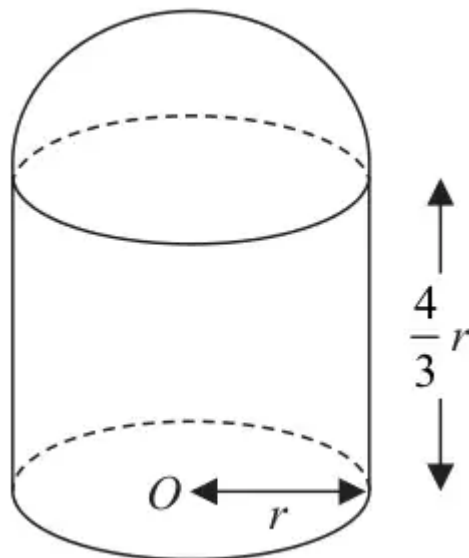


Figure 3

A uniform solid cylinder of base radius r and height $\frac{4}{3}r$ has the same density as a uniform solid hemisphere of radius r . The plane face of the hemisphere is joined to a plane face of the cylinder to form the composite solid S shown in Figure 3. The point O is the centre of the plane face of S .

- (a) Show that the distance from O to the centre of mass of S is $\frac{73}{72}r$. (4)

The solid S is placed with its plane face on a rough horizontal plane. The coefficient of friction between S and the plane is μ . A horizontal force P is applied to the highest point of S . The magnitude of P is gradually increased.

- (b) Find the range of values of μ for which S will slide before it starts to tilt. (5)

(Total 9 marks)

3.

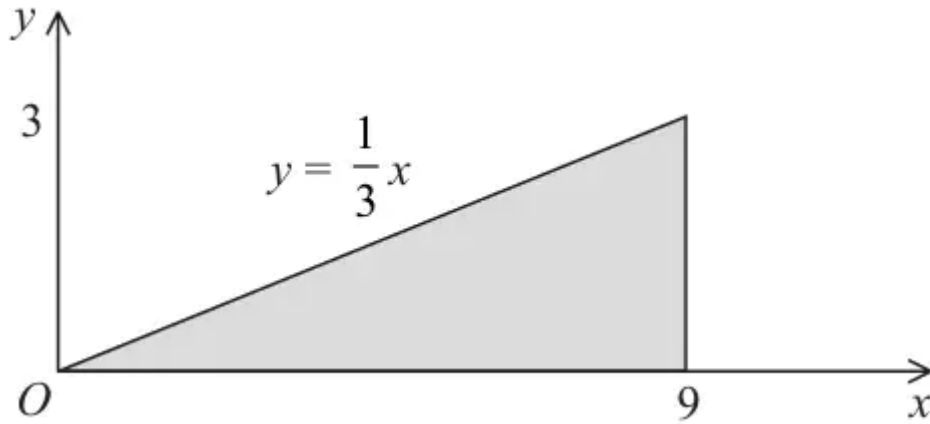


Figure 4

The shaded region shown in Figure 4 is bounded by the x -axis, the line with equation $x = 9$ and the line with equation $y = \frac{1}{3}x$. This shaded region is rotated through 360° about the x -axis to form a solid of revolution. This solid of revolution is used to model a solid right circular cone of height 9 cm and base radius 3 cm.

The cone is non-uniform and the mass per unit volume of the cone at the point (x, y, z) is $\lambda x \text{ kg cm}^{-3}$, where $0 \leq x \leq 9$ and λ is constant.

(a) Find the distance of the centre of mass of the cone from its vertex. (6)

A toy is made by joining the circular plane face of the cone to the circular plane face of a uniform solid hemisphere of radius 3 cm, so that the centres of the two plane surfaces coincide.

The weight of the cone is W newtons and the weight of the hemisphere is kW newtons.

When the toy is placed on a smooth horizontal plane with any point of the curved surface of the hemisphere in contact with the plane, the toy will remain at rest.

(b) Find the value of k (4)

(Total 10 marks)

Solutions

1.

Scheme	Marks
Use of $(\pi) \int y^2 dx \times \bar{x} = (\pi) \int xy^2 dx$	M1
$\int x dx \times \bar{x} = \int x^2 dx$	
$\left[\frac{1}{2}x^2\right] \dots \times \bar{x} = \left[\frac{1}{3}x^3\right] \dots$	A1 = A1
Using limits 0 and 4 $\frac{16}{2} \times \bar{x} = \frac{64}{3}$	M1
$\bar{x} = \frac{8}{3}$	A1
	(5)
	(5 marks)

2.

Mark scheme (a)

Scheme	Marks	A0
$\frac{4}{3}\pi r^3 \times \frac{2}{3}r + \frac{2}{3}\pi r^3 \times \left(\frac{4}{3}r + \frac{3}{8}r\right) = \left(\frac{4}{3} + \frac{2}{3}\right)\pi r^3 \times d$	M1	2.1
$\left(\frac{8}{9}r + \frac{8}{9}r + \frac{1}{4}r = 2d\right)$ $\left(\frac{73}{36}r = 2d\right)$	A1 A1	1.1b 1.1b
$\Rightarrow d = \frac{73}{72}r$ *	A1*	2.2a
	(4)	

Notes

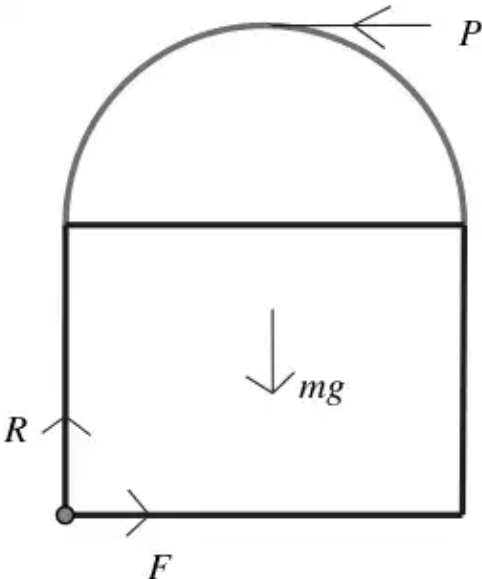
M1: Moments equation. Dimensionally correct.

A1: Unsimplified equation with at most one slip

A1: Correct unsimplified equation

A1*: Obtain given result from correct working.

Mark scheme (b)

Scheme	Marks	AO
		
Resolving: $\leftrightarrow F = P, \uparrow R = Mg, F_{\max} = \mu R = \mu Mg$	M1	1.1b
Slides if $P > \mu Mg$	A1	1.2
Moments: $\frac{7}{3}rP = rMg$ Tilts if $P > \frac{3}{7}Mg$	B1	1.1b
Comparison of restrictions to determine values of μ	M1	3.1a
Slides first if $\mu Mg < \frac{3}{7}Mg, \quad (0 <) \mu < \frac{3}{7}$	A1	2.2a
	(5)	
	(9 marks)	

Notes

M1: Resolve and use $F = \mu R$ to find values of P for sliding

A1: Use the model to form the correct inequality

B1: Correct inequality for tilting

M1: Correct comparison of when it tilts and when it slides

A1: Correct conclusion

3.

Mark scheme (a)

Scheme	Marks	AO
Mass of cone = $\int_0^9 \pi y^2 \lambda x \, dx = \pi \lambda \int_0^9 \frac{x^3}{9} \, dx$	M1	3.4
$= \pi \lambda \left[\frac{x^4}{36} \right]_0^9 \quad \left(= \frac{729\pi\lambda}{4} \text{ (kg)} \right)$	A1	1.1b
Moments: $\int_0^9 \pi y^2 \lambda x \times x \, dx = \pi \lambda \int_0^9 \frac{x^4}{9} \, dx$	M1	3.4
$= \frac{\pi\lambda}{45} [x^5]_0^9 \quad \left(= \frac{\pi\lambda}{5} \times 9^4 \right)$	A1	1.1b
$\Rightarrow d = \frac{\frac{\pi\lambda}{5} \times 9^4}{\frac{\pi\lambda}{4} \times 9^3}$	DM1	2.1
$d = \frac{36}{5} = 7.2 \text{ (cm)}$	A1	1.1b
	(6)	

Notes

NB: Some candidates are confusing the mass and the volume. For the first M1A1:

- If they have a correct method for the mass and they tell you that this is mass, award the marks.
- If they have a correct method for the mass say nothing, but use it correctly, award the marks.
- If they have a correct method for the mass, say nothing, and use it as the moment, then M0 because this implies that they do not think it is the mass.

M1: Use the model to find the mass of the cone. Allow without limits.

A1: Correct integration. Correct limits seen or implied

Substitution not required.

Allow 2/2 if π not seen and consistent with (b) if attempted

M1: Use the model to find the moment of the cone (usual rules for integration)

Allow without limits

A1: Correct integration. Correct limits seen or implied

Substitution not required.

Allow 2/2 if π not seen and consistent with (a)

M1: Complete method to find the distance of the centre of mass from the vertex. A complete method requires the two preceding M marks.

They need to get as far as a value for d .

If they have a method that comes directly to this stage you might not see the λ or π

A1: Correct only

If all you see is $\Rightarrow d = \frac{9^5}{45} \div \frac{9^4}{36}$ or even $\Rightarrow d = \frac{9}{5} \times 4$ then award 6/6

Allow 6/6 if π not seen throughout but otherwise correct

Mark scheme (b)

Scheme	Marks	AO
Remains at rest $\Rightarrow$ centre of mass at centre of plane surface	B1	2.1
Moments about diameter of plane surface:	M1	3.1b
$(9 - d)W \left\{ = \left(9 - \frac{36}{5} \right) W \right\} = \frac{3}{8} \times 3 \times kW$	A1ft	1.1b
$k = \frac{8}{5}$	A1	1.1b
	(4)	
	(10 marks)	

Notes

B1: Correct deduction for location of c of m

Stated or implied by their moments equation

M1: Moments about diameter of plane face(s)

M0 if the moments equation contradicts the centre of mass being on the interface

M0 if using volume in place of mass

A1ft: Correct unsimplified equation. Follow their 7.2

Alternative moments equations:

Using vertex: $W\bar{x} + kW \left(9 + \frac{3}{8} \times 3 \right) = (W + kW) \times 9$

Using base: $W(12 - \bar{x}) + kW \left(3 - 3 \times \frac{3}{8} \right) = (W + kW)3$

If they are working with the axis at an angle they will possibly have trig terms which should cancel.

A1: Correct only