

QQQ – Further Mechanics 2 – Chapter 2 – Centres of Mass of Plane Figures (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Three particles of masses $4m$, $2m$ and km are placed at the points with coordinates $(-3, -1)$, $(6, 1)$ and $(-1, 5)$ respectively.

Given that the centre of mass of the three particles is at the point with coordinates $(\bar{x}, \bar{y})$

(a) show that $\bar{x} = \frac{-k}{k+6}$ (3)

(b) find $\bar{y}$ in terms of k . (2)

Given that the centre of mass of the three particles lies on the line with equation $y = 2x + 3$

(c) find the value of k . (2)

A fourth particle is placed at the point with coordinates $(\lambda, 4)$.

Given that the centre of mass of the four particles also lies on the line with equation $y = 2x + 3$

(d) find the value of λ . (2)

(Total 9 marks)

2.

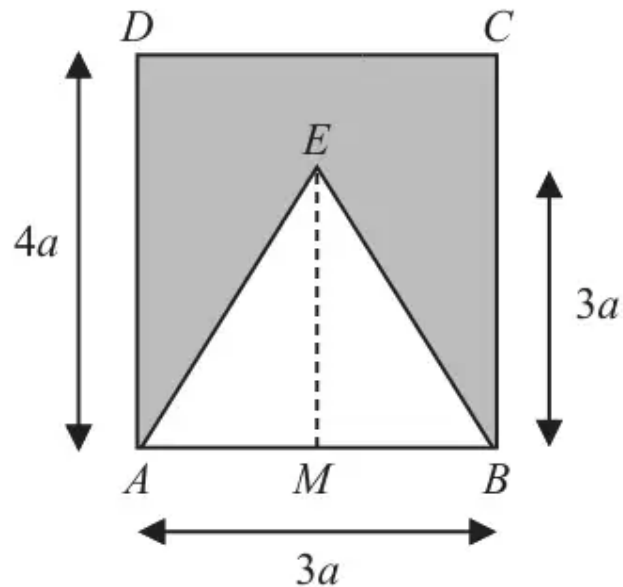


Figure 1

A uniform plane lamina, shown shaded in Figure 1, is formed by removing an isosceles triangle ABE from a rectangle $ABCD$.

- The midpoint of AB is M
- $AB = 3a$
- $AD = 4a$
- $EM = 3a$
- $AE = EB$

- (a) Show that the distance of the centre of mass of the lamina from AB is $\frac{13a}{5}$ (5)

The lamina is suspended by a string attached to the lamina at D .

The lamina hangs freely in equilibrium.

- (b) Find, to the nearest degree, the angle between AD and the vertical. (3)

(Total 8 marks)

3.

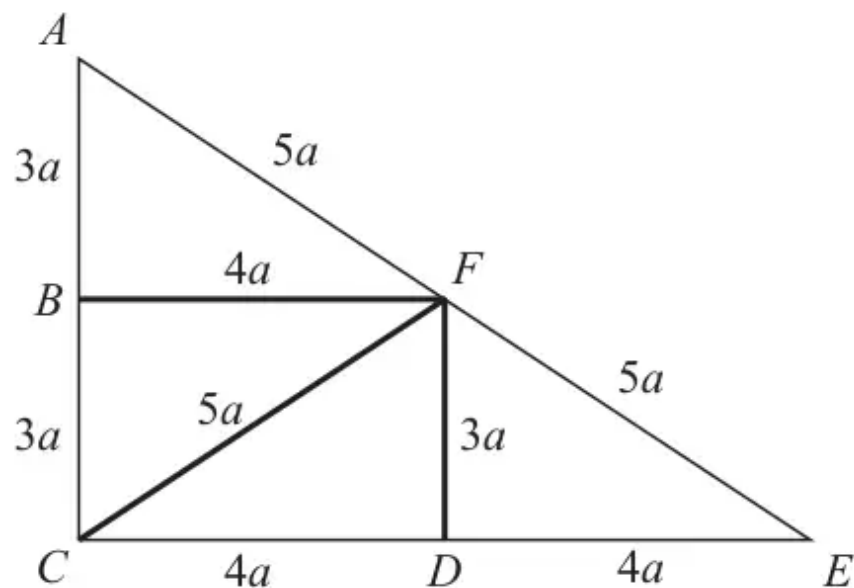


Figure 1

Nine uniform rods are joined together to form the rigid framework $ABCDEF A$, with $AB = BC = DF = 3a$, $BF = CD = DE = 4a$ and $AF = FE = CF = 5a$, as shown in Figure 1. All nine rods lie in the same plane.

The mass per unit length of each of the rods BF , CF and DF is twice the mass per unit length of each of the other six rods.

(a) Find the distance of the centre of mass of the framework from AC (4)

The mass of the framework is M . A particle of mass kM is attached to the framework at E to form a loaded framework.

When the loaded framework is freely suspended from F , it hangs in equilibrium with CE horizontal.

(b) Find the exact value of k (3)

(Total 7 marks)