

QQQ – Further Mechanics 2 – Chapter 2 – Centres of Mass of Plane Figures (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Three particles of masses $4m$, $2m$ and km are placed at the points with coordinates $(-3, -1)$, $(6, 1)$ and $(-1, 5)$ respectively.

Given that the centre of mass of the three particles is at the point with coordinates $(\bar{x}, \bar{y})$

(a) show that $\bar{x} = \frac{-k}{k+6}$ (3)

(b) find $\bar{y}$ in terms of k . (2)

Given that the centre of mass of the three particles lies on the line with equation $y = 2x + 3$

(c) find the value of k . (2)

A fourth particle is placed at the point with coordinates $(\lambda, 4)$.

Given that the centre of mass of the four particles also lies on the line with equation $y = 2x + 3$

(d) find the value of λ . (2)

(Total 9 marks)

2.

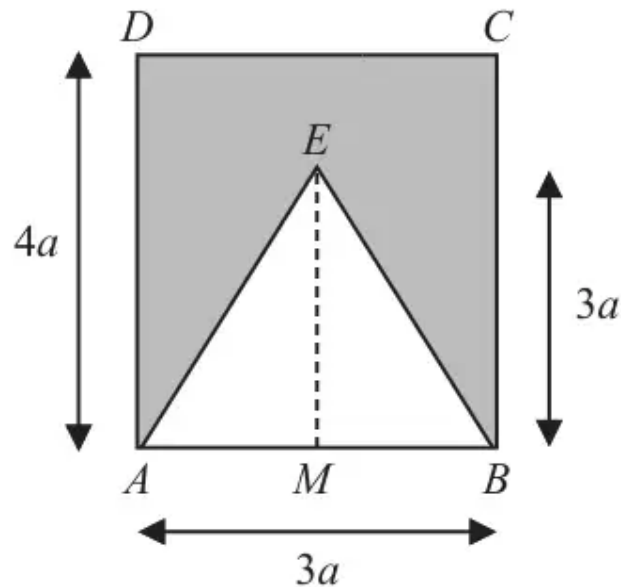


Figure 1

A uniform plane lamina, shown shaded in Figure 1, is formed by removing an isosceles triangle ABE from a rectangle $ABCD$.

- The midpoint of AB is M
- $AB = 3a$
- $AD = 4a$
- $EM = 3a$
- $AE = EB$

- (a) Show that the distance of the centre of mass of the lamina from AB is $\frac{13a}{5}$ (5)

The lamina is suspended by a string attached to the lamina at D .

The lamina hangs freely in equilibrium.

- (b) Find, to the nearest degree, the angle between AD and the vertical. (3)

(Total 8 marks)

3.

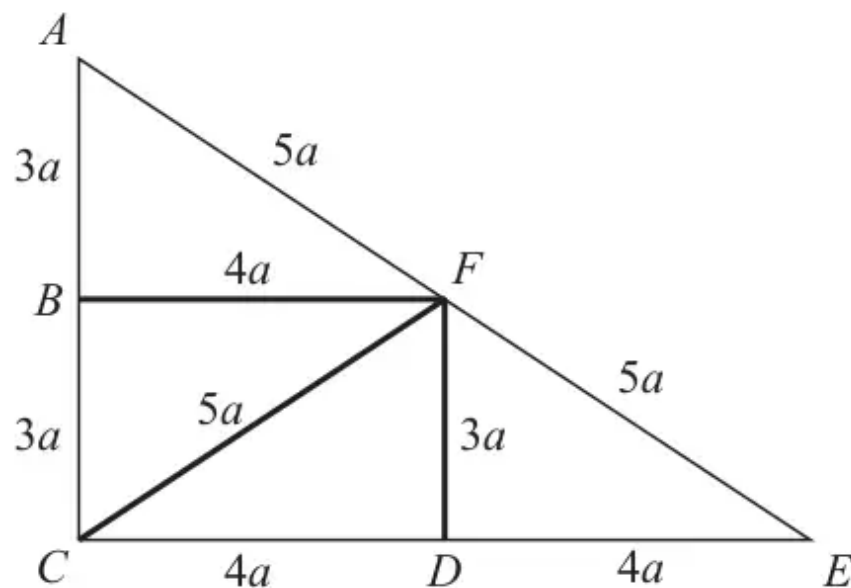


Figure 1

Nine uniform rods are joined together to form the rigid framework $ABCDEF A$, with $AB = BC = DF = 3a$, $BF = CD = DE = 4a$ and $AF = FE = CF = 5a$, as shown in Figure 1. All nine rods lie in the same plane.

The mass per unit length of each of the rods BF , CF and DF is twice the mass per unit length of each of the other six rods.

(a) Find the distance of the centre of mass of the framework from AC (4)

The mass of the framework is M . A particle of mass kM is attached to the framework at E to form a loaded framework.

When the loaded framework is freely suspended from F , it hangs in equilibrium with CE horizontal.

(b) Find the exact value of k (3)

(Total 7 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
Moments about y axis:	M1	2.1
$km \times -1 + 4m \times -3 + 2m \times 6 = (k + 6)m \times \bar{x}$	A1	1.1b
$\Rightarrow \bar{x} = \frac{-k}{k + 6} *$	A1*	1.1b
	(3)	

Notes

M1: Moments about y axis (or a parallel axis). All terms required. Dimensionally consistent. Condone sign errors.

A1: Correct unsimplified equation

A1*: Obtain given answer from full and correct working

Accept with $6 + k$

Mark scheme (b)

Scheme	Marks	AO
Moments about x axis: $(km \times 5 + 4m \times -1 + 2m \times 1 = (k + 6)m \times \bar{y})$	M1	3.1a
$\Rightarrow \bar{y} = \frac{5k - 2}{k + 6}$	A1	1.1b
	(2)	

Notes

M1: Moments about x axis (or a parallel axis). All terms required. Dimensionally consistent. Condone sign errors.

A1: Correct unsimplified expression for $\bar{y}$.

Any equivalent simplified form

The first 5 marks are available for a combined equation in vector form

Mark scheme (c)

Scheme	Marks	AO
$\bar{y} = 2\bar{x} + 3 \Rightarrow \frac{5k - 2}{k + 6} = \frac{-2k}{k + 6} + 3$ $(5k - 2 = -2k + (3k + 18))$	M1	1.1b
$\Rightarrow k = 5$	A1	1.1b
	(2)	

Notes

M1: Correct use of their $\bar{y}$ and given $\bar{x}$ to find k .

A1: Correct only

Mark scheme (d)

Scheme	Marks	AO
$4 = 2\lambda + 3 \Rightarrow$	M1	3.4
$\lambda = \frac{1}{2}$	A1	1.1b
	(2)	
	(9 marks)	

Notes

M1: Use the model to obtain $\lambda = \dots$ or $x = \dots$

If working from the beginning and the new particle has mass M then

$$\frac{23 + 4M}{11 + M} = 2 \left(\frac{-5 + \lambda M}{11 + M} \right) + 3$$

A1: Accept $x = \frac{1}{2}$

2.

Mark scheme (a)

Scheme	Marks	AO
$ABCD$ ABE lamina		

$12a^2 \quad \frac{9a^2}{2} \quad \frac{15a^2}{2}$	B1	1.2
$2a \quad a \quad \bar{x}$	B1	1.2
Moments about AB	M1	2.1
$(12a^2 \times 2a) - \left(\frac{9a^2}{2} \times a \right) = \frac{15a^2}{2} \bar{x}$	A1	1.1b
$\bar{x} = \frac{13a}{5} *$	A1*	2.2a
	(5)	

Notes

B1: Any equivalent ratios

B1: Or correct distances from a parallel axis

M1: Or moments about a parallel axis

A1: Correct unsimplified equation for their axis

A1*: Correct given answer correctly obtained. Condone $\frac{13}{5}a$

Mark scheme (b)

Scheme	Marks	A0
Use of trigonometry to find a relevant angle	M1	3.1a
$\tan \alpha = \frac{\frac{3a}{2}}{4a - \frac{13a}{5}}$	A1	1.1b
$\alpha = 47^\circ$ or 133° (nearest degree)	A1	1.1b
	(3)	
	(8 marks)	

Notes

M1: A complete method to obtain angle with e.g. vertical or horizontal

A1: Correct unsimplified equation (or its reciprocal)

A1: cao

3.

Mark scheme (a)

Scheme										Marks	AO
Moments about AC :										M1	3.1a
rod	CD	DE	EF	FA	AB	BC	BF	DF	CF		
Mass ratio	4	4	5	5	3	3	8	6	10		
From AC	$2a$	$6a$	$6a$	$2a$	0	0	$2a$	$4a$	$2a$		
$8a \times 4a + 2 \times 3a \times 4a + 2 \times 5a \times 2a + 10a \times 4a + 2 \times 4a \times 2a = 48a\bar{x}$										A1 A1	1.1b 1.1b
$(132a = 48\bar{x}) \Rightarrow \bar{x} = \frac{11}{4}a$										A1	1.1b
										(4)	

Notes

M1: Dimensionally correct equation for moments about AC or a parallel axis. **All terms** needed and horizontal distances

Must be using the mass ratio. Allow slips, but not consistently density and not consistently lengths. Condone without a

A1: One side of the equation correct

A1: Both sides of the equation correct

A1: Or equivalent single term

Condone if a is missing in the working and appears at the end

Mark scheme (b)

Scheme	Marks	AO
Moments about F :	M1	3.1a

$Mg(4a - \bar{x}) = kMg \times 4a$	A1ft	1.1b
$\Rightarrow k = \frac{5}{16}$	A1	1.1b
	(3)	
	(7 marks)	

Notes

M1: Dimensionally correct moments equation. Accept any complete alternative method using M and kM to obtain an equation in k only.

Condone if g and / or M cancelled throughout

Condone incorrect distances

Condone if use $M = 48$ throughout

A1ft: Correct unsimplified equation (accept without g and/or M) Correct mass and distance combination for their $\bar{x}$

A1: Or 0.3125 Condone 0.31 or 0.313

Alternative

Scheme	Marks	AO
Moments about C :	M1	
$4a(M + kM) = M\bar{x} + 8akM$	A1ft	
$\Rightarrow k = \frac{5}{16}$	A1	1.1b
	(3)	