

QQQ – Further Mechanics 2 – Chapter 2 – Centres of Mass of Plane Figures (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Three particles of masses $3m$, $4m$ and km are positioned at the points with coordinates $(2a, 3a)$, $(a, 5a)$ and $(2\mu a, \mu a)$ respectively, where k and μ are constants.

The centre of mass of the three particles is at the point with coordinates $(2a, 4a)$.

Find

(i) the value of k

(ii) the value of μ

(6)

(Total 6 marks)

2.

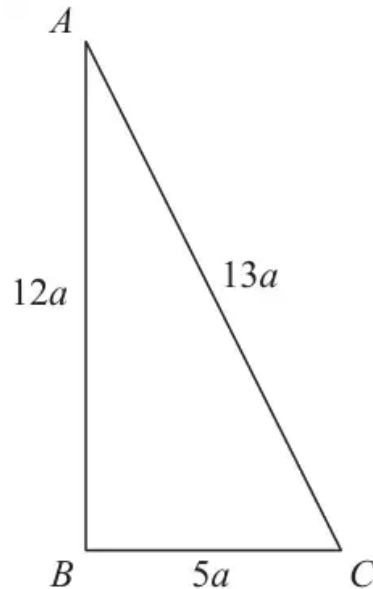


Figure 1

A thin uniform rod, of total length $30a$ and mass M , is bent to form a frame. The frame is in the shape of a triangle ABC , where $AB = 12a$, $BC = 5a$ and $CA = 13a$, as shown in Figure 1.

- (a) Show that the centre of mass of the frame is $\frac{3}{2}a$ from AB . (4)

The frame is freely suspended from A . A horizontal force of magnitude kMg , where k is a constant, is applied to the frame at B . The line of action of the force lies in the vertical plane containing the frame. The frame hangs in equilibrium with AB vertical.

- (b) Find the value of k . (3)

(Total 7 marks)

3.

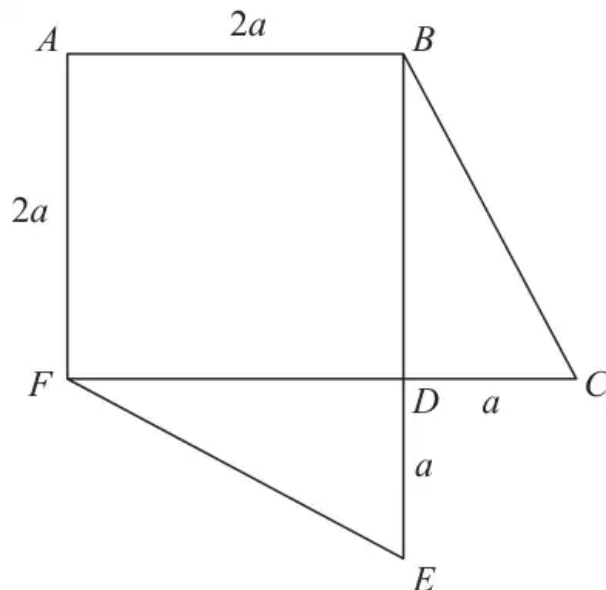


Figure 2

The lamina L , shown in Figure 2, consists of a uniform square lamina $ABDF$ and two uniform triangular laminas BDC and FDE . The square has sides of length $2a$. The two triangles are identical.

The straight lines BDE and FDC are perpendicular with $BD = DF = 2a$ and $DC = DE = a$.

The mass per unit of area of the square is M .

The mass per unit area of each triangle is $3M$.

The centre of mass of L is at the point G .

(a) Without doing any calculations, explain why G lies on AD . (1)

(b) Show that the distance of G from D is $\frac{\sqrt{2}}{2}a$ (7)

The lamina L is freely suspended from B and hangs in equilibrium.

(c) Find the size of the angle between BE and the downward vertical. (3)

(Total 11 marks)