

QQQ – Further Mechanics 2 – Chapter 2 – Centres of Mass of Plane Figures (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. Three particles of masses $3m$, $4m$ and km are positioned at the points with coordinates $(2a, 3a)$, $(a, 5a)$ and $(2\mu a, \mu a)$ respectively, where k and μ are constants.

The centre of mass of the three particles is at the point with coordinates $(2a, 4a)$.

Find

(i) the value of k

(ii) the value of μ

(6)

(Total 6 marks)

2.

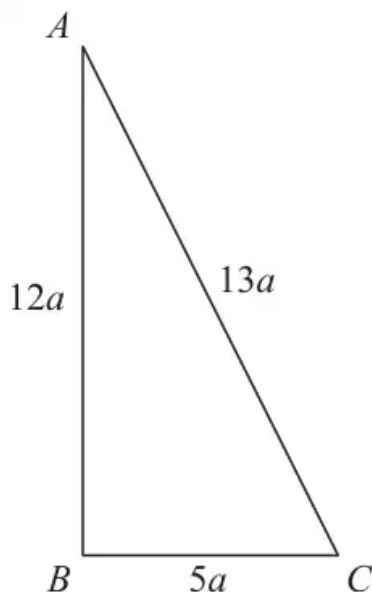


Figure 1

A thin uniform rod, of total length $30a$ and mass M , is bent to form a frame. The frame is in the shape of a triangle ABC , where $AB = 12a$, $BC = 5a$ and $CA = 13a$, as shown in Figure 1.

- (a) Show that the centre of mass of the frame is $\frac{3}{2}a$ from AB . (4)

The frame is freely suspended from A . A horizontal force of magnitude kMg , where k is a constant, is applied to the frame at B . The line of action of the force lies in the vertical plane containing the frame. The frame hangs in equilibrium with AB vertical.

- (b) Find the value of k . (3)

(Total 7 marks)

3.

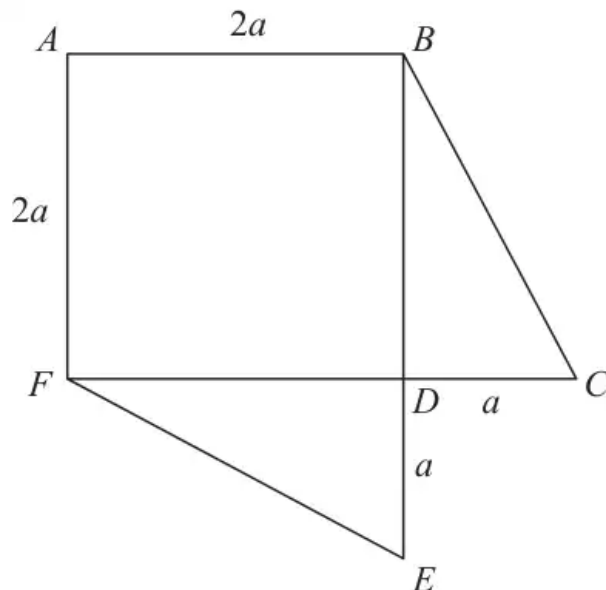


Figure 2

The lamina L , shown in Figure 2, consists of a uniform square lamina $ABDF$ and two uniform triangular laminas BDC and FDE . The square has sides of length $2a$. The two triangles are identical.

The straight lines BDE and FDC are perpendicular with $BD = DF = 2a$ and $DC = DE = a$.

The mass per unit of area of the square is M .

The mass per unit area of each triangle is $3M$.

The centre of mass of L is at the point G .

(a) Without doing any calculations, explain why G lies on AD . (1)

(b) Show that the distance of G from D is $\frac{\sqrt{2}}{2}a$. (7)

The lamina L is freely suspended from B and hangs in equilibrium.

(c) Find the size of the angle between BE and the downward vertical. (3)

(Total 11 marks)

Solutions

1.

Scheme	Marks	AO
Moments about y -axis	M1	3.1a
$2a(7 + k) = 3 \times 2a + 4 \times a + k \times 2\mu a$ $(14 + 2k = 10 + 2\mu k)$	A1	1.1b
Moments about x -axis	M1	3.1a
$4a(7 + k) = 3 \times 3a + 4 \times 5a + k \times \mu a$ $(28 + 4k = 29 + \mu k)$	A1	1.1b
Obtain $k = 1$ or $\mu = 3$	A1	2.2a
Obtain $k = 1$ and $\mu = 3$	A1	2.2a
	(6)	
	(6 marks)	

Notes

M1: Dimensionally correct equation with all terms. Allow with $\bar{x}$ in place of $2a$

A1: Correct unsimplified equation.

M1: Dimensionally correct equation with all terms. Allow with $\bar{y}$ in place of $4a$

A1: Correct unsimplified equation.

These two equations can be combined as a single vector equation.

A1: One correct value.

A1: Both correct values.

2.

Mark scheme (a)

Scheme	Marks	AO
Complete strategy to find d	M1	3.1b

$\frac{5}{30}M \times \frac{5}{2}a + \frac{13}{30}M \times \frac{5}{2}a = M \times d$	A1	1.1b
$\left(\frac{25}{2}a + \frac{65}{2}a = 30d \right)$	A1	1.1b
$90a = 60d \Rightarrow d = \frac{3}{2}a^*$	A1*	2.1
	(4)	

Notes

M1: Complete strategy to find d e.g. moments about AB or a parallel axis. Needs all relevant terms. Must be dimensionally correct.

Condone sign errors. M 's might cancel from the start.

A1: Unsimplified equation with at most one error

A1: Correct unsimplified equation

A1*: Obtain the **given answer** from a convincing argument

Mark scheme (b)

Scheme	Marks	AO
Complete strategy to find k , e.g. by use of a moments equation	M1	3.1b
$Mg \times \frac{3}{2}a = kMg \times 12a$	A1	1.1b
$k = \frac{1}{8}$	A1	1.1b
	(3)	
	(7 marks)	

Notes

M1: Complete strategy to find k e.g. moments about A .

Needs all relevant terms. Must be dimensionally correct.

Condone sign errors. Condone if a, M, g missing throughout

A1: Correct unsimplified equation in k

A1: Correct answer – any equivalent form

Alternative (b)

Scheme	Marks	AO
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Moments equation	M1	
$12a \times kM = \frac{13}{30}M \times 2.5a + \frac{5}{30}M \times 2.5a$	A1	
$12k = \frac{45}{30}, \quad k = \frac{1}{8}$	A1	
	(3)	

3.

Mark scheme (a)

Scheme	Marks	AO
L is symmetrical about AD	B1	2.4
	(1)	

Notes

B1: Any equivalent statement about the symmetry

Mark scheme (b)

Scheme					Marks	AO
	$ABDF$	BCD	DEF	L		
Mass ratio	$4a^2 \times M$	$a^2 \times 3M$	$a^2 \times 3M$	$10a^2 \times M$		
C of M from BE	$-a$	$+\frac{a}{3}$	$-\frac{2a}{3}$	x		
Mass ratios					B1	1.2
Distances from BE					B1	1.2
Moments equation					M1	2.1
$-a \times 4a^2M + \frac{a}{3} \times 3a^2M - \frac{2a}{3} \times 3a^2M = 10a^2M \times x$ $(-4a + a - 2a = 10x)$					A1	1.1b
$x = -\frac{5a}{10} = -\frac{a}{2}$					A1	1.1b

Use symmetry and Pythagoras	M1	1.1a
Distance from $D = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{\sqrt{2}}{2}a$	A1*	2.2a
	(7)	

Notes

B1: Correct mass ratios

B1: Distance ratios from any horizontal or vertical axis

M1: Moments equation for complete lamina about any horizontal or vertical axis. Must be dimensionally correct

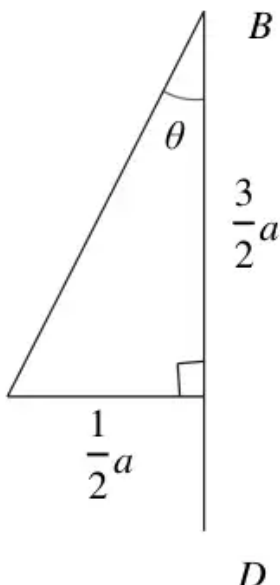
A1: Correct unsimplified equation for their axes

A1: Correct horizontal or vertical distance from D

M1: Use of Pythagoras with their distance

A1*: Obtain given answer from correct working.

Mark scheme (c)

Scheme	Marks	AO
		
Trig ratio of a relevant angle	M1	1.2
$\tan \theta = \frac{1}{3} \text{ or } \cos \theta = \frac{\frac{10}{4}a^2 + 4a^2 - \frac{2}{4}a^2}{2 \times \frac{\sqrt{10}}{2}a \times 2a} = \frac{6}{2\sqrt{10}}$	A1ft	1.1b

$\theta = 18.4^\circ$	A1	1.1b
	(3)	
	(11 marks)	

Notes

M1: Trig ratio of θ or $90^\circ - \theta$ or equivalent

A1ft: Correct unsimplified expression using their $\frac{a}{2}$

A1: Correct angle. Accept 0.322 radians