

QQQ – Further Mechanics 2 – Chapter 1 – Circular Motion (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1.

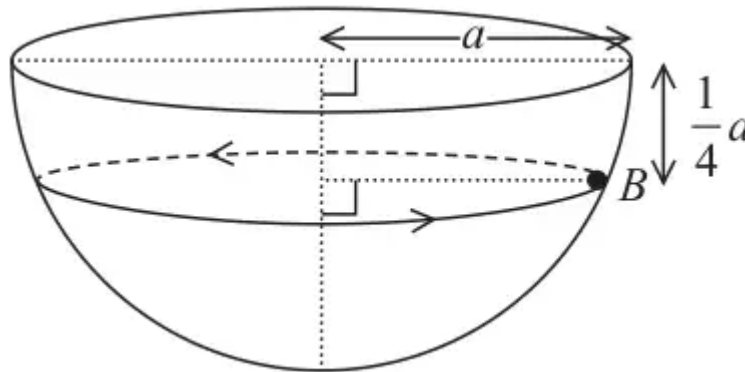


Figure 1

A hemispherical shell of radius a is fixed with its rim uppermost and horizontal. A small bead, B , is moving with constant angular speed, ω , in a horizontal circle on the smooth inner surface of the shell. The centre of the path of B is at a distance $\frac{1}{4}a$ vertically below the level of the rim of the hemisphere, as shown in Figure 1.

Find the magnitude of ω , giving your answer in terms of a and g .

(6)

(Total 6 marks)

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2. A rough disc rotates about its centre in a horizontal plane with constant angular speed 80 revolutions per minute. A particle P lies on the disc at a distance 8 cm from the centre of the disc. The coefficient of friction between P and the disc is μ . Given that P remains at rest relative to the disc, find the least possible value of μ .

(7)

(Total 7 marks)

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3. A fixed smooth sphere has centre O and radius a . A particle P is placed on the surface of the sphere at the point A , where OA makes an angle α with the upward vertical through O . The particle is released from rest at A . When OP makes an angle θ to the upward vertical through O , P is on the surface of the sphere and the speed of P is v .

Given that $\cos \alpha = \frac{3}{5}$

(a) show that

$$v^2 = \frac{2ga}{5}(3 - 5 \cos \theta)$$

(4)

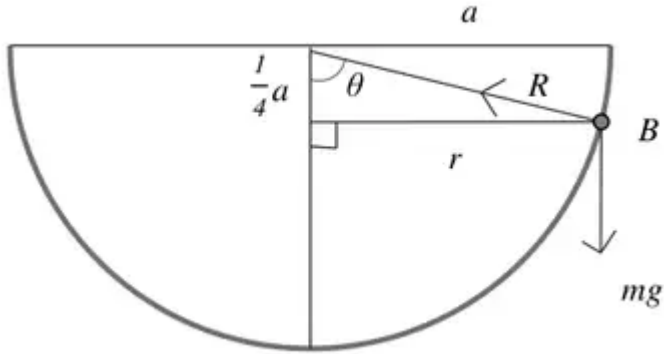
(b) find the speed of P at the instant when it loses contact with the sphere.

(8)

(Total 12 marks)

Solutions

1.

Scheme	Marks	AO
		
$\updownarrow R \cos \theta = mg$	M1	3.1b
$\leftrightarrow R \sin \theta = mr\omega^2$	M1	3.3
	A1	1.1b
$\tan \theta = \frac{r}{a/4} \quad (\tan \theta = \sqrt{15})$	B1	1.1b
Complete strategy to find ω	M1	3.1b
$\tan \theta = \frac{mr\omega^2}{mg} = \frac{4r}{a}, \quad \Rightarrow \omega^2 = \frac{4g}{a}, \quad \omega = 2\sqrt{\frac{g}{a}}$	A1	1.1b
	(6)	
	(6 marks)	

Notes

Check their diagram to see where they have put θ

Check the working carefully, particularly the value of r : some errors in the working can lead to a fortuitously correct answer

M1: Resolve vertically. Must be dimensionally correct.

M1: Resolve horizontally and form equation for circular motion. Must be dimensionally correct.

A1: Correct pair of equations for their unknowns (any r)

B1: Correct trig ratio(s) seen or implied. Allow for $r = \frac{\sqrt{15}}{4}a$

M1: Complete strategy to form and solve a set of equations with $r \neq a$ to find ω . For solving their 2 equations – not dependent

A1: Eliminate additional variables to obtain ω . Accept equivalent exact forms.

If R does not act through the centre of the hemisphere then the maximum available is M1M1A0B0M0A0:
2/6

Alternative

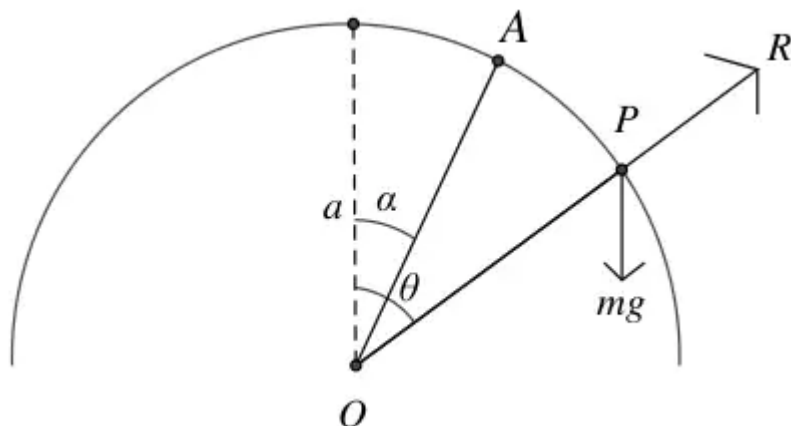
Scheme	Marks	AO
$\uparrow R \cos \theta = mg$	M1	3.1b
$\leftrightarrow R \sin \theta = ma \sin \theta \omega^2$	M1	3.3
	A1	1.1b
$\cos \theta = \frac{1}{4}$	B1	1.1b
Complete strategy to find ω	M1	3.1b
$\Rightarrow R = 4mg, \quad 4mg = ma\omega^2 \quad \Rightarrow \quad \omega = 2\sqrt{\frac{g}{a}}$	A1	1.1b

2.

Scheme	Marks
$\omega = \frac{80 \times 2\pi}{60} \text{ rad s}^{-1} \left(= \frac{8\pi}{3} \approx 8.377 \dots \right)$ Accept $v = \frac{16\pi}{75} \approx 0.67 \text{ ms}^{-1}$ as equivalent	B1
$(\uparrow) R = mg$	B1
For least value of μ $(\leftarrow) \mu mg = mr\omega^2$	M1 A1=A1
$\mu = \frac{0.08}{9.8} \times \left(\frac{8\pi}{3} \right)^2 \approx 0.57$ accept 0.573	M1 A1
	(7)
	(7 marks)

3.

Mark scheme (a)



Scheme	Marks
Conservation of energy : Loss in GPE = gain in KE	M1
$mga (\cos \alpha - \cos \theta) = \frac{1}{2}mv^2$	A2,1,0
Substitute for $\cos \alpha$ and rearrange to given answer :	A1
$v^2 = \frac{2mga}{m} \left(\frac{3}{5} - \cos \theta \right) = \frac{2ga}{5} (3 - 5 \cos \theta) *$	
	(4)

Mark scheme (b)

Scheme	Marks
Considering the acceleration towards the centre of the hemisphere:	M1
$mg \cos \theta - R = \frac{mv^2}{a}$	A2,1,0
Substitute for v^2 to form expression for R :	DM1
$R = mg \cos \theta - \frac{mv^2}{a} = mg (3 \cos \theta - 2 \cos \alpha) \left(= mg \left(3 \cos \theta - \frac{6}{5} \right) \right)$	A1
Loses contact with the surface when $R = 0$	M1
$\cos \theta = \frac{2}{5}$	A1
$v^2 = \frac{2ga}{5}, \quad v = \sqrt{\frac{2ga}{5}}$	A1
	(8)
	(12 marks)

Alt:

$R = 0 \Rightarrow mg \cos \theta = \frac{mv^2}{a}$	DM1
$\cos \theta = \frac{v^2}{ga}$	A1
Substitute in given (a) $v^2 = \frac{2ga}{5} \left(3 - 5 \frac{v^2}{ga} \right)$	M1
$v^2 = \frac{6ga}{5} - 2v^2, \quad 3v^2 = \frac{6ga}{5}$	A1
$v = \sqrt{\frac{2ga}{5}}$	A1