

QQQ – Further Mechanics 2 – Chapter 1 – Circular Motion (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1.

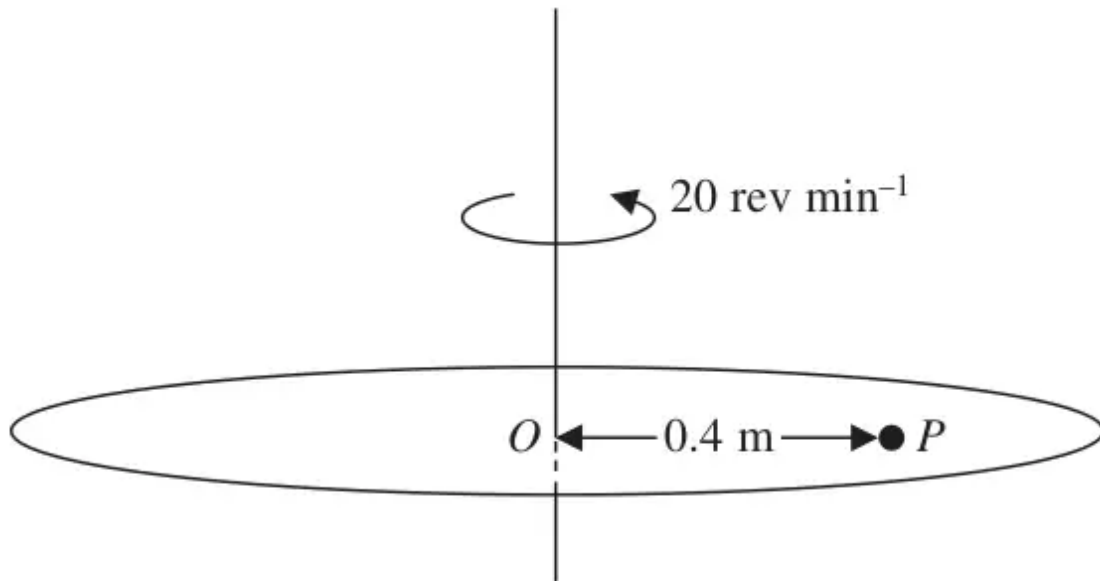


Figure 1

A rough disc is rotating in a horizontal plane with constant angular speed 20 revolutions per minute about a fixed vertical axis through its centre O . A particle P rests on the disc at a distance 0.4 m from O , as shown in Figure 1. The coefficient of friction between P and the disc is μ . The particle P is on the point of slipping.

Find the value of μ .

(6)

(Total 6 marks)

2. A car moves round a bend which is banked at a constant angle of θ° to the horizontal.

When the car is travelling at a constant speed of 80 km h^{-1} there is no sideways frictional force on the car. The car is modelled as a particle moving in a horizontal circle of radius 500 m.

(a) Find the value of θ . **(7)**

(b) Identify one limitation of this model. **(1)**

The speed of the car is increased so that it is now travelling at a constant speed of 90 km h^{-1}

The car is still modelled as a particle moving in a horizontal circle of radius 500 m.

(c) Describe the extra force that will now be acting on the car, stating the direction of this force. **(1)**

(Total 9 marks)

3.

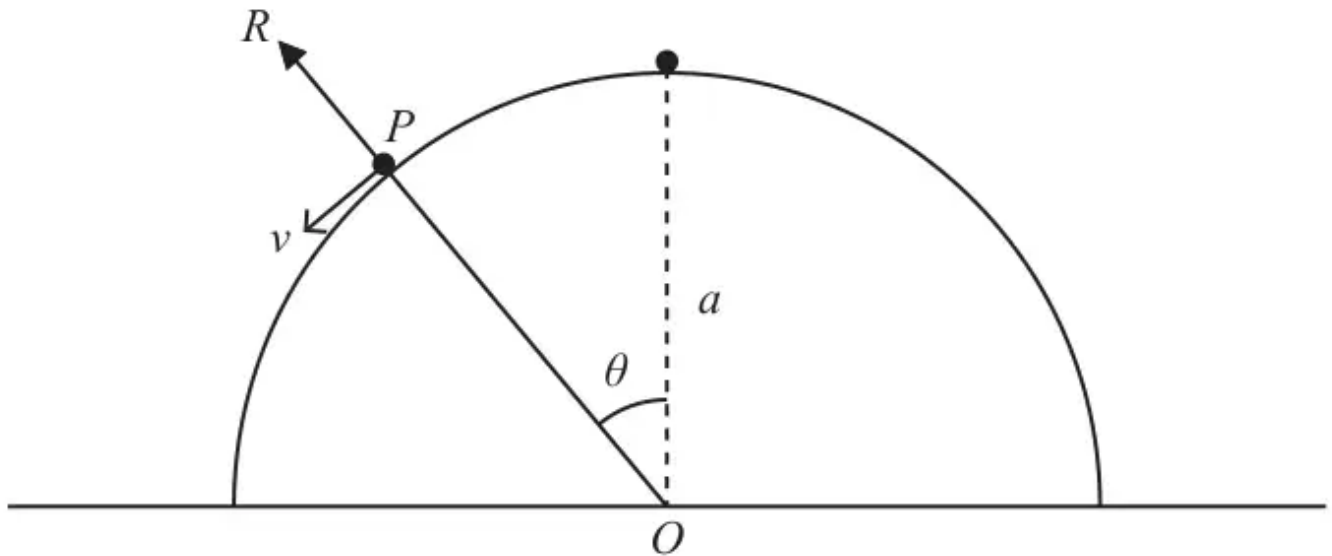


Figure 2

A smooth hemisphere of radius a is fixed on a horizontal surface with its plane face in contact with the surface. The centre of the plane face of the hemisphere is O .

A particle P of mass M is disturbed from rest at the highest point of the hemisphere.

When P is still on the surface of the hemisphere and the radius from O to P is at an angle θ to the vertical,

- the speed of P is v
- the normal reaction between the hemisphere and the particle is R , as shown in Figure 2.

(a) Show that $R = Mg(3 \cos \theta - 2)$ (6)

(b) Find, in terms of a and g , the speed of the particle at the instant when the particle leaves the surface of the hemisphere. (3)

(Total 9 marks)

Solutions

1.

Scheme	Marks
$R(\uparrow) \ R = mg$	
$F = \mu mg$	B1
$20 \text{ revs per min} = \frac{20}{60} \times 2\pi \text{ rad s}^{-1} \left(= \frac{2}{3}\pi \text{ rad s}^{-1} \right)$	M1A1
$R(\rightarrow) \ \mu mg = m \times 0.4 \times \left(\frac{2}{3}\pi \right)^2$	M1A1ft
$\mu = \frac{0.4 \times 4\pi^2}{9g}$	
$\mu = 0.18 \text{ or } 0.179$	A1
	(6 marks)

Notes

B1 for resolving vertically and using $F = \mu R$ to obtain $F = \mu mg$. This may not be seen explicitly, but give B1 when seen used in an equation.

M1 for attempting to change revs per minute to rad s^{-1} , must see $(2)\pi$. (Can use 60 or 60^2)

A1 for $\frac{20}{60} \times 2\pi \text{ (rad s}^{-1}\text{) oe}$

M1 for NL2 horizontally along the radius - acceleration in either form for this mark, F or μmg or μm all allowed. r to be 0.4 now or later. This is not dependent on the previous M mark.

A1ft for $\mu mg = m \times 0.4 \times \left(\frac{2}{3}\pi \right)^2$ follow through on their ω

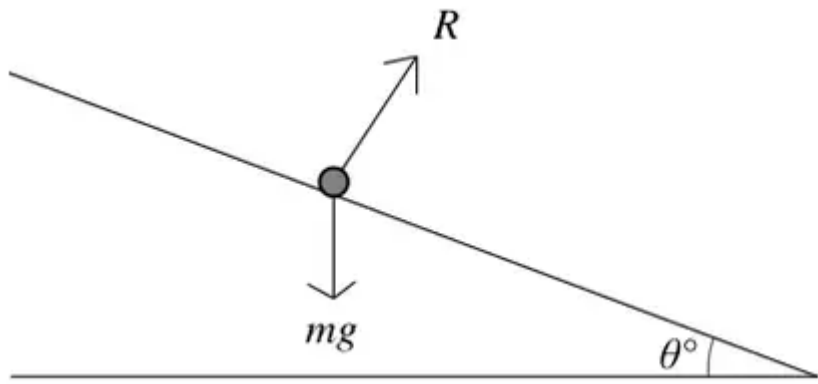
A1cso for $\mu = 0.18 \text{ or } 0.179$, **must be 2 or 3 sf.**

NB: Use of $\leq$: is allowed, provided used correctly, until the final statement, which must be $\mu = \dots$

2.

Mark scheme (a)

Scheme	Marks	AO
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Complete strategy to find value of θ	M1	3.1b
		
Resolve vertically	M1	3.1b
$R \cos \theta^\circ = mg$	A1	1.1b
Resolve horizontally	M1	3.1b
$R \sin \theta^\circ = \frac{mv^2}{r}$	A1	1.1b
$v = 80 \text{ km h}^{-1} = \frac{80 \times 1000}{60^2} \text{ m s}^{-1}$	B1	1.2
Solve simultaneous equations and substitute v in correct units to obtain θ : $\tan \theta^\circ = \frac{v^2}{rg} = \frac{640000}{36^2 \times 500 \times 9.8}, \quad \theta = 5.8$	A1	2.2a
	(7)	

Notes

M1: Complete strategy involving resolving in perpendicular directions, change of units and solution of simultaneous equations

M1: Complete strategy to form one equation involving θ e.g. resolve vertically. Condone sin/cos confusion

A1: Or equivalent

M1: Complete strategy to form a second equation involving θ e.g. resolve horizontally. Condone sin/cos confusion

A1: Correct unsimplified – need not substitute for v or r

B1: Correct conversion km h^{-1} to m s^{-1} (22.2)

A1: Accept 5.8 or 5.75 (follows use of 9.8)

Mark scheme (b)

Scheme	Marks	AO
All weight acting at a single point	B1	3.5b
	(1)	

Notes

B1: Any appropriate comment

e.g. Only one point of contact with the road

The centre of mass of the car is on the road.

Mark scheme (c)

Scheme	Marks	AO
Friction acting down the slope	B1	2.2a
	(1)	
	(9 marks)	

Notes

B1: Need to include the direction

3.

Mark scheme (a)

Scheme	Marks	AO
Conservation of energy:	M1	3.1a
$\frac{1}{2}Mv^2 = Mga(1 - \cos \theta)$	A1	1.1b
Equation of motion:	M1	3.1a
$\frac{Mv^2}{a} = Mg \cos \theta - R$	A1	1.1b
Solve for R :	DM1	2.1
$R = Mg \cos \theta - \frac{Mv^2}{a} = Mg \cos \theta - 2Mg(1 - \cos \theta)$ $= 3Mg \cos \theta - 2Mg = Mg(3 \cos \theta - 2) \quad *$	A1*	2.2a

	(6)	
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Notes

M1: Dimensionally correct equation. All relevant terms. Condone sign error(s) and sin/cos confusion.

A1: Correct unsimplified equation.

M1: Dimensionally correct equation. All relevant terms. Condone sign error(s) and sin/cos confusion.

A1: Correct unsimplified equation.

If they have more than 2 equations, mark the correct equations. If they go on to use an incorrect equation then DM0.

DM1: Complete method to obtain expression for R

A1*: Obtain given answer from full and correct working.

Mark scheme (b)

Scheme	Marks	AO
$R = 0 \Rightarrow \cos \theta = \frac{2}{3}$	B1	1.1b
$v^2 = 2ga(1 - \cos \theta)$	M1	3.1a
$v^2 = \frac{2}{3}ga, \quad v = \sqrt{\frac{2ga}{3}}$	A1	1.1b
	(3)	
	(9 marks)	

Notes

B1: Seen or implied.

M1: Complete method to obtain v or v^2

A1: Any equivalent form.

Allow $0.82\sqrt{ga}$ or better.