

QQQ – Further Mechanics 1 – Chapter 5 – Collisions in Two Dimensions (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. Two small smooth spheres A and B have equal radii. The mass of A is $2m$ kg and the mass of B is m kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of A is $(2\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$ and the velocity of B is $(-3\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$. Immediately after the collision the velocity of A is $(\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$.

Find the speed of B immediately after the collision.

(5)

(Total 5 marks)

Edexcel M4 June 2008 Q2

2. [In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

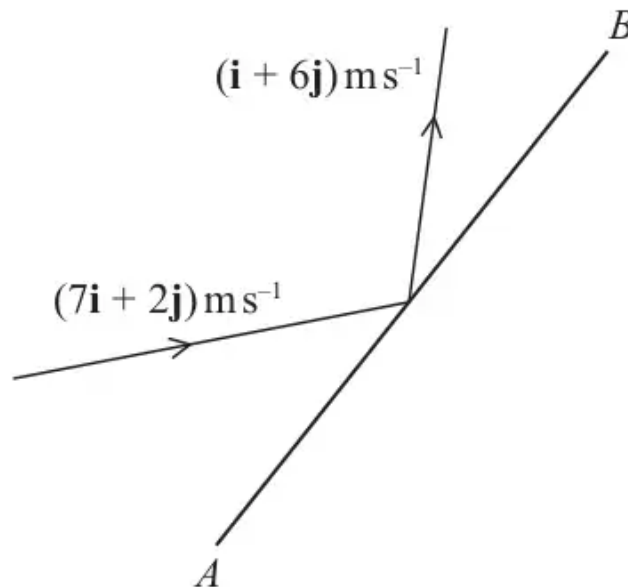


Figure 1

Figure 1 represents the plan view of part of a smooth horizontal floor, where AB represents a fixed smooth vertical wall.

A small ball of mass 0.5 kg is moving on the floor when it strikes the wall.

Immediately before the impact the velocity of the ball is $(7\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$.

Immediately after the impact the velocity of the ball is $(\mathbf{i} + 6\mathbf{j}) \text{ m s}^{-1}$.

The coefficient of restitution between the ball and the wall is e .

(a) Show that AB is parallel to $(2\mathbf{i} + 3\mathbf{j})$. (4)

(b) Find the value of e . (5)

(Total 9 marks)

3.

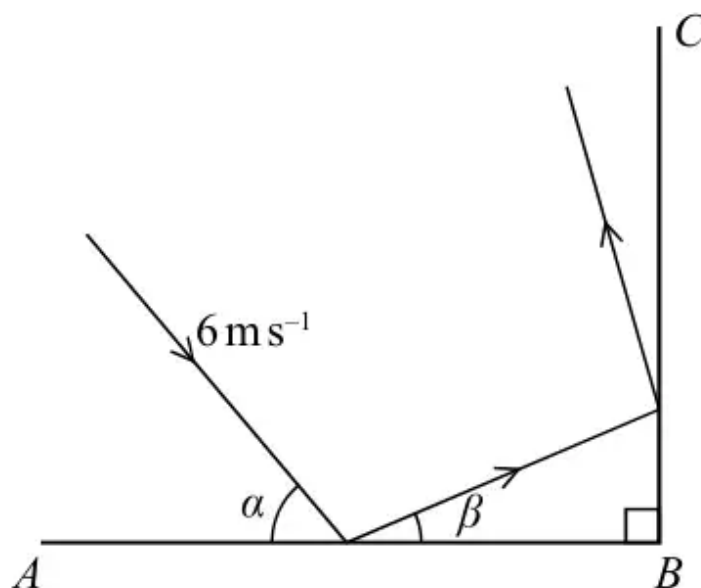


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where AB and BC are fixed vertical walls with AB perpendicular to BC .

A small ball is projected along the floor towards AB with speed 6 m s^{-1} on a path that makes an angle α with AB , where $\tan \alpha = \frac{4}{3}$. The ball hits AB and then hits BC .

Immediately after hitting AB , the ball is moving at an angle β to AB , where $\tan \beta = \frac{1}{3}$

The coefficient of restitution between the ball and AB is e .

The coefficient of restitution between the ball and BC is $\frac{1}{2}$

By modelling the ball as a particle and the floor and walls as being smooth,

- (a) show that the value of $e = \frac{1}{4}$ (5)
- (b) find the speed of the ball immediately after it hits BC . (4)
- (c) Suggest two ways in which the model could be refined to make it more realistic. (2)

(Total 11 marks)

Solutions

1.

Scheme	Marks
$2m(2\mathbf{i} - 2\mathbf{j}) + m(-3\mathbf{i} - \mathbf{j}) = 2m(\mathbf{i} - 3\mathbf{j}) + m\mathbf{v}$	M1 A1
$(\mathbf{i} - 5\mathbf{j}) = (2\mathbf{i} - 6\mathbf{j}) + \mathbf{v}$	
$(-\mathbf{i} + \mathbf{j}) = \mathbf{v}$	A1
$ \mathbf{v} = \sqrt{(-1)^2 + 1^2} = \sqrt{2} \text{ m s}^{-1} \text{ cwo}$	DM1 A1
	(5 marks)

2.

Mark scheme (a)

Scheme	Marks	AO
Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ or $\mathbf{v} - \mathbf{u}$	M1	2.1
$\mathbf{I} = 0.5((\mathbf{i} + 6\mathbf{j}) - (7\mathbf{i} + 2\mathbf{j})) \quad (= (-3\mathbf{i} + 2\mathbf{j}))$	A1	1.1b
Use of scalar product $(-3\mathbf{i} + 2\mathbf{j}) \cdot (2\mathbf{i} + 3\mathbf{j}) = -6 + 6 = 0$	M1	1.1b
Hence impulse perpendicular to $(2\mathbf{i} + 3\mathbf{j})$, so AB must be parallel to $(2\mathbf{i} + 3\mathbf{j})$. *	A1*	2.2a
	(4)	

Notes

Correction: In Alternative (a) 2 the ratio is $b = \frac{3}{2}a$ (corrected from the printed mark scheme: $b = \frac{2}{3}a$).

M1: Must be finding the difference between two momenta or two velocities

A1: Correct unsimplified equation for the impulse or for change in velocity

M1: Use of scalar product or equivalent. In the alt method allow full marks if $\sqrt{13}$ not used.

A1*: Reach given conclusion from correct working

If working with angles, score

M1 for correct method to find components parallel to the wall

A1 for $\sqrt{53} \cos 40.36^\circ$ and $\sqrt{37} \cos 24.23^\circ$

M1 for comparing the two values

A0 because the work has involved decimal approximations (since working towards an exact given answer).

Alternative: Could use $e \tan 24.2^\circ = \tan 40.36^\circ$

Alternative (a)

Scheme	Marks	AO
Components of velocities parallel to $(2\mathbf{i} + 3\mathbf{j})$:	M1	2.1
$\left(\frac{1}{\sqrt{13}}\right)(7\mathbf{i} + 2\mathbf{j}) \cdot (2\mathbf{i} + 3\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(14 + 6)$ $\left(\frac{1}{\sqrt{13}}\right)(\mathbf{i} + 6\mathbf{j}) \cdot (2\mathbf{i} + 3\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(2 + 18)$	A1	1.1b
Simplify and compare values	M1	1.1b
Hence component of velocity parallel to $(2\mathbf{i} + 3\mathbf{j})$ is unchanged, so AB must be parallel to $(2\mathbf{i} + 3\mathbf{j})$. *	A1*	2.2a
	(4)	

Alternative (a) 2

Scheme	Marks	AO
Use conservation of velocity parallel to $a\mathbf{i} + b\mathbf{j}$	M1	2.1
$(7\mathbf{i} + 2\mathbf{j}) \cdot (a\mathbf{i} + b\mathbf{j}) = (\mathbf{i} + 6\mathbf{j}) \cdot (a\mathbf{i} + b\mathbf{j})$ $(\Rightarrow 7a + 2b = a + 6b)$	A1	1.1b
Find ratio of a and b to obtain direction: $\left(b = \frac{3}{2}a\right)$	M1	1.1b
Hence AB must be parallel to $(2\mathbf{i} + 3\mathbf{j})$. *	A1*	2.2a
	(4)	

Mark scheme (b)

Scheme	Marks	AO
Use scalar product to find components of velocities perpendicular to the wall	M1	3.1b

$\left(\frac{1}{\sqrt{13}}\right)(-3\mathbf{i} + 2\mathbf{j}) \cdot (7\mathbf{i} + 2\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(-21 +$ 4) $\left(= \frac{-17}{\sqrt{13}}\right)$ $\left(\frac{1}{\sqrt{13}}\right)(-3\mathbf{i} + 2\mathbf{j}) \cdot (\mathbf{i} + 6\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(-3 +$ 12) $\left(= \frac{9}{\sqrt{13}}\right)$	A1 A1	1.1b 1.1b
Use of impact law	M1	3.4
$e = \frac{9}{17}$	A1	1.1b
	(5)	
	(9 marks)	

Notes

M1: Condone if not using a unit vector

A1 A1: One correct value

Second correct values

If working with angles, score M1A1A1 for $\sqrt{53} \sin 40.36..^\circ$ and $\sqrt{37} \sin 24.23...^\circ$

M1: Use their components the right way round in the impact law. Condone sign error.

A1: 0.53 or better (0.52941.....)

3.

Mark scheme (a)

Scheme	Marks	AO
After hit AB : $\rightarrow 6 \cos \alpha (= v \cos \beta) (= 3.6)$	B1	3.1b
Use of impact law:	M1	3.4
$\uparrow 6e \sin \alpha (= v \sin \beta) \left(= \frac{24e}{5}\right) (= 4.8e)$	A1	1.1b
$\tan \beta = \frac{1}{3} = \frac{6e \sin \alpha}{6 \cos \alpha} \left(= \frac{24e}{5} \div \frac{18}{5}\right)$	M1	2.1
$e = \frac{18}{3 \times 24} = \frac{1}{4}^*$	A1*	2.2a
	(5)	

Notes

B1: Use model to find component parallel to the wall

M1: Use model and impact law perpendicular to the wall

A1: Correct perpendicular component

M1: Use $\frac{1}{3}$ and their components to form equation in e $\left(v = \frac{6\sqrt{10}}{5} = 3.79\right)$

A1*: Correct answer from correct exact working

If only see $e \tan \alpha = \tan \beta$ with no explanation of where it comes from then score 0/5

Mark scheme (b)

Scheme	Marks	AO
After hit BC : $\uparrow \frac{6}{5}$	B1	1.1b
$\rightarrow \frac{1}{2} \times \frac{18}{5} \left(= \frac{9}{5}\right)$	B1	3.4
Speed = $\frac{3}{5} \sqrt{2^2 + 3^2}$	M1	1.1b
$= \frac{3\sqrt{13}}{5} \text{ (m s}^{-1}\text{)}$	A1	1.1b
	(4)	

Notes

B1: First component correct

B1: Second component correct

Alternative: B1 for speed of impact with $BC = 3.79 \dots$

B1 for path on leaving BC at $56.3 \dots^\circ$ to BC

M1: Use Pythagoras' theorem or trigonometry to find the speed

A1: Any equivalent form. 2.2 or better (2.1633...)

Mark scheme (c)

Scheme	Marks	AO
An appropriate refinement	B1	3.5c

A second independent appropriate refinement and no incorrect refinements	B1	3.5c
	(2)	
	(11 marks)	

Notes

B1 B1: Two independent refinements relating to the modelling e.g.

- Include friction between the floor and the ball
- Include friction between the ball and the walls
- Give the ball dimensions.
- Consider air resistance
- Spin / rotation

Do not accept comments about mass / gravity / levels / perpendicularity