

QQQ – Further Mechanics 1 – Chapter 5 – Collisions in Two Dimensions (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1.

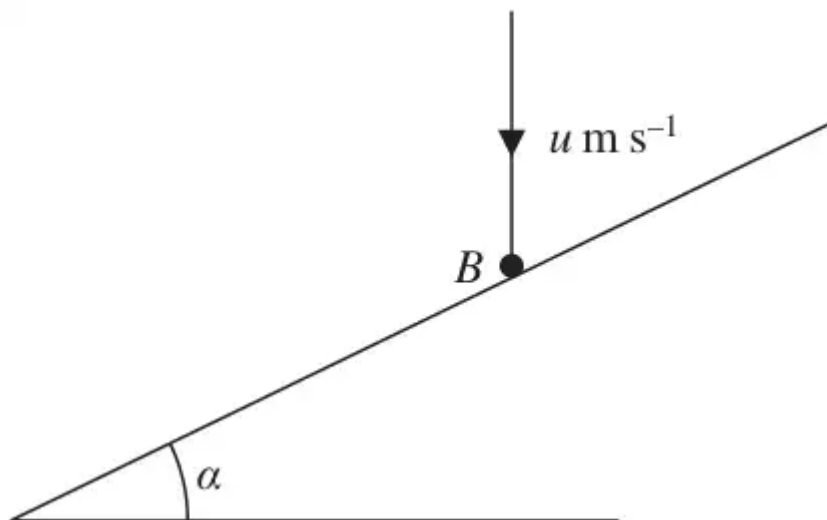


Figure 1

A smooth fixed plane is inclined at an angle α to the horizontal. A smooth ball B falls vertically and hits the plane. Immediately before the impact the speed of B is $u \text{ m s}^{-1}$, as shown in Figure 1. Immediately after the impact the direction of motion of B is horizontal. The coefficient of restitution between B and the plane is $\frac{1}{3}$.

Find the size of angle α .

(6)

(Total 6 marks)

2.

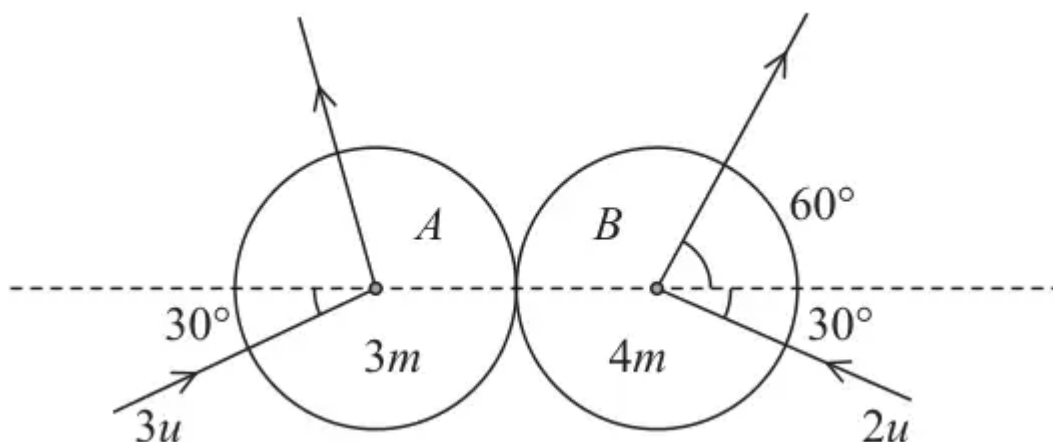


Figure 3

Two smooth uniform spheres, A and B , have equal radii. The mass of A is $3m$ and the mass of B is $4m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before they collide, A is moving with speed $3u$ at 30° to the line of centres of the spheres and B is moving with speed $2u$ at 30° to the line of centres of the spheres. The direction of motion of B is turned through an angle of 90° by the collision, as shown in Figure 3.

- (i) Find the size of the angle through which the direction of motion of A is turned as a result of the collision.
- (ii) Find, in terms of m and u , the magnitude of the impulse received by B in the collision.

(9)

(Total 9 marks)

3.

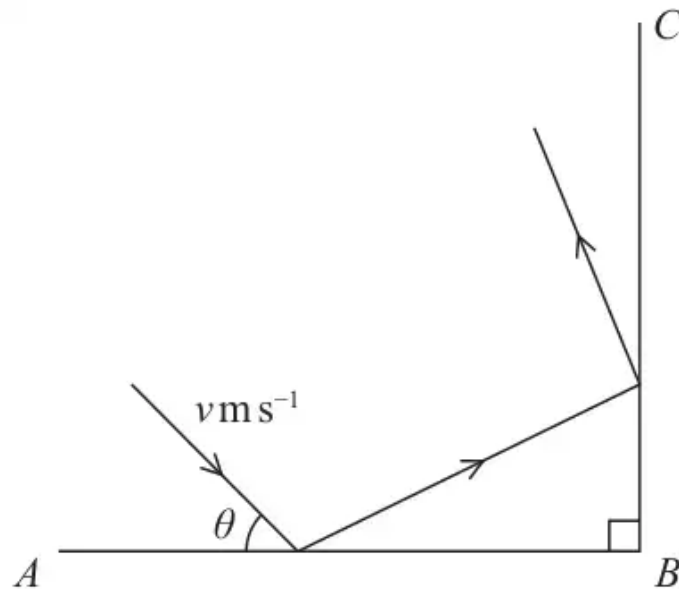


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls, with AB perpendicular to BC .

A small ball is projected along the floor towards the wall AB . Immediately before hitting the wall AB the ball is moving with speed $v \text{ m s}^{-1}$ at an angle θ to AB .

The ball hits the wall AB and then hits the wall BC .

The coefficient of restitution between the ball and the wall AB is $\frac{1}{3}$.

The coefficient of restitution between the ball and the wall BC is e .

The floor and the walls are modelled as being smooth.

The ball is modelled as a particle.

The ball loses half of its kinetic energy in the impact with the wall AB .

(a) Find the exact value of $\cos \theta$. (5)

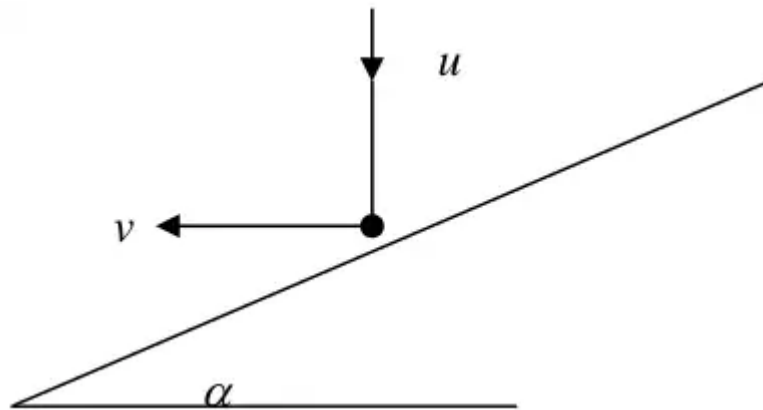
The ball loses half of its remaining kinetic energy in the impact with the wall BC .

(b) Find the exact value of e . (5)

(Total 10 marks)

Solutions

1.



Scheme	Marks
CLM: $u \sin \alpha = v \cos \alpha$	M1 A1
Impact: $\frac{1}{3}u \cos \alpha = v \sin \alpha$	M1 A1
$\frac{1}{3} \times \frac{1}{\tan \alpha} = \tan \alpha$	M1
$\tan \alpha = \frac{1}{\sqrt{3}}$	
$\alpha = 30^\circ$ (or $\frac{\pi}{6}$ or 0.52 rad)	A1
	(6)
	(6 marks)

Notes

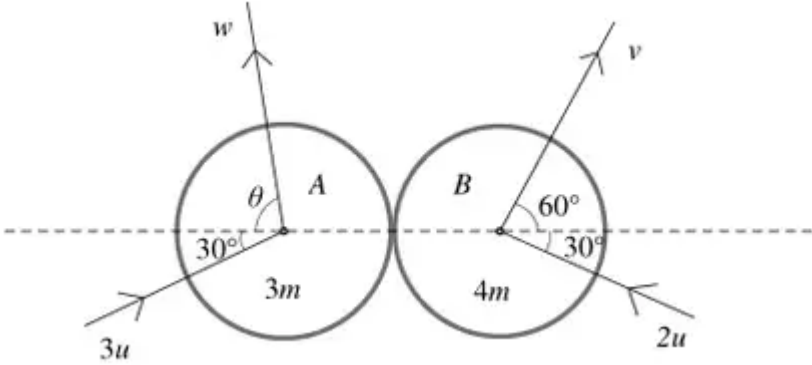
M1 A1 Must be in correct direction but condone trig confusion

M1 A1 Condone consistent trig confusion

2.

Mark scheme (i)

Scheme	Marks	AO
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Parallel to line of centres:	M1	3.1b
$9mu \cos 30^\circ - 8mu \cos 30^\circ = 4mv \cos 60^\circ - 3mw \cos \theta$ $(u \cos 30^\circ = 2v - 3w \cos \theta) \quad (u \cos 30^\circ = 2v - 3w_x)$	A1	1.1b
$\updownarrow A : (w_y =) w \sin \theta = 3u \sin 30^\circ \left(= \frac{3u}{2} \right)$ $\updownarrow B : v \sin 60^\circ = 2u \sin 30^\circ (= u) \quad \left(v = \frac{2u}{\sqrt{3}} \right)$	B1 B1	3.4 3.4
$(w_x =) w \cos \theta = \frac{1}{3}(2v - u \cos 30^\circ) = \frac{5u\sqrt{3}}{18}$ $(w_y =) w \sin \theta = \frac{3u}{2}$ $\left(\Rightarrow \tan \theta = \frac{9\sqrt{3}}{5}, \theta = 72.2^\circ \right)$	M1	1.1b
Direction deflected by 77.8° (78° or better)	A1	2.2a

Notes

M1: Use of CLM parallel to the line of centres.

Need all 4 terms. Dimensionally correct. Condone sign errors and sin/cos confusion.

A1: Correct unsimplified equation. Allow e.g. w_x in place of $w \cos \theta$ and v_x in place of $v \cos 60^\circ$.

Allow if they have divided through by a common factor e.g. m

NB there is no mark for the correct use of the impact law because the candidates are not required to find the coefficient of restitution. They might however find it as part of an alternative method. In this case, the M marks below are for a complete correct method to achieve the required result. Ignore work to find e if it is not used.

B1 B1: No change perpendicular to line of centres for one sphere. Allow e.g. w_y in place of $w \sin \theta$.

Check the diagrams – the vertical components are often shown there.

No change perpendicular to line of centres for both spheres

M1: Use scalar product or solve simultaneous equations to find θ for a relevant angle using their w_x

They need to get as far as $\theta =$ a numerical value for a relevant angle

A1: 78° or better

Mark scheme (ii)

Scheme	Marks	AO
Magnitude of impulse	M1	3.1b
$= 4m(v \cos 60^\circ - (-2u \cos 30^\circ))$	A1	1.1b
$= 4m \left(\frac{1}{2} \times \frac{u}{\sin 60^\circ} - \left(-2u \frac{\sqrt{3}}{2} \right) \right) = \frac{16\sqrt{3}}{3}mu$	A1	2.2a
OR: magnitude $= 3m(3u \cos 30^\circ + w \cos \theta)$ $= 3m \left(+\frac{5u\sqrt{3}}{18} - \left(-\frac{3u\sqrt{3}}{2} \right) \right) = \frac{16\sqrt{3}}{3}mu$		
	(9)	
	(9 marks)	

Notes

M1: Use of $I = mv - mu$ in direction of line of centres. Condone subtraction in either order

Allow M1 if they think that they have subtracted but they have not actually taken account of the change of direction.

Allow M1 if they go direct to the correct expression with a + without telling you that they have taken account of the change in direction

Allow M1 if they go straight to an unsimplified expression in surds using values already found earlier.

A1: Correct unsimplified expression.

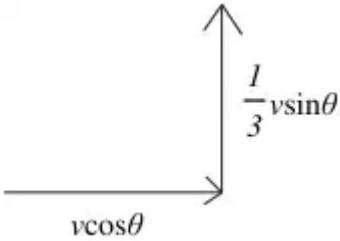
Allow the negative of this

A1: Any equivalent simplified form. Must be positive. Condone if they change sign at the very end without explaining why. Accept $9.2(376 \dots)mu$ (2 sf or better)

NB You might see candidates using the right angle and matrix multiplication to rotate the initial velocity of B to find the correct components of the velocity of B after impact.

3.

Mark scheme (a)

Scheme	Marks	AO
Use the model to find components of velocity after first impact:	B1 B1	1.1b 3.4
		
Kinetic energy: $\frac{1}{2} \times \frac{1}{2} m v^2 = \frac{1}{2} m \left(v^2 \cos^2 \theta + \frac{1}{9} v^2 \sin^2 \theta \right)$	M1	3.1b
$\frac{1}{2} = \frac{1}{9} + \frac{8}{9} \cos^2 \theta$	M1	1.1b
$\frac{7}{16} = \cos^2 \theta, \quad \cos \theta = \frac{\sqrt{7}}{4}$	A1	1.1b
	(5)	

Notes

B1 B1: Parallel component correct

Perpendicular component correct Check the diagram

M1: Equation for KE in v, θ . Dimensionally correct. Includes all components. Condone $\frac{1}{2}$ used on wrong side

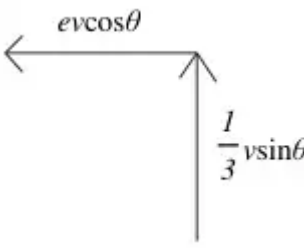
M1: Form and solve equation in $\cos \theta$

A1: Or exact equivalent

Alternative (a)

Scheme	Marks	AO
Working with initial velocity $\mathbf{v} = x\mathbf{i} - y\mathbf{j}$, after impact $\mathbf{v} = x\mathbf{i} + \frac{1}{3}y\mathbf{j}$	B1 B1	1.1b 3.4
KE: $\frac{1}{2} \times \frac{1}{2} m (x^2 + y^2) = \frac{1}{2} m \left(x^2 + \frac{1}{9} y^2 \right)$	M1	3.1b
$y^2 = \frac{9}{7} x^2, \quad \frac{y}{x} = \tan \theta = \frac{3}{\sqrt{7}}$	M1	1.1b
$\cos \theta = \frac{\sqrt{7}}{4}$	A1	1.1b
	(5)	

Mark scheme (b)

Scheme	Marks	AO
Use the model to find components of velocity after second impact: 	B1 B1	1.1b 3.4
Kinetic energy: $\frac{1}{4} \times \frac{1}{2}mv^2 = \frac{1}{2}m \left(e^2v^2 \cos^2 \theta + \frac{1}{9}v^2 \sin^2 \theta \right)$ or $\frac{1}{2} \times \frac{1}{2}m \left(v^2 \cos^2 \theta + \frac{1}{9}v^2 \sin^2 \theta \right) =$ $\frac{1}{2}m \left(e^2v^2 \cos^2 \theta + \frac{1}{9}v^2 \sin^2 \theta \right)$	M1	3.1b
$\frac{1}{4} = \frac{7}{16}e^2 + \frac{1}{9} \times \frac{9}{16}$	M1	1.1b
$\Rightarrow e^2 = \frac{3}{7}, \quad e = \sqrt{\frac{3}{7}}$	A1	1.1b
	(5)	
	(10 marks)	

Notes

B1 B1: Parallel component correct

Perpendicular component correct

M1: Equation for KE in x, y . Dimensionally correct. Includes all components. Condone $\frac{1}{2}$ used on wrong side

M1: Use their $\cos \theta$ to form and solve equation in e

A1: Or exact equivalent

Alternative (b)

Scheme	Marks	AO
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After second impact $\mathbf{v} = -ex\mathbf{i} + \frac{1}{3}y\mathbf{j}$	B1 B1	1.1b 3.4
KE: $\frac{1}{4} \times \frac{1}{2}m(x^2 + y^2) = \frac{1}{2}m\left(e^2x^2 + \frac{1}{9}y^2\right)$	M1	3.1b
$4e^2x^2 + \frac{4}{9}y^2 = x^2 + y^2, \quad 4e^2 = 1 + \frac{5}{9}\left(\frac{y}{x}\right)^2$	M1	1.1b
$\Rightarrow e^2 = \frac{3}{7}, \quad e = \sqrt{\frac{3}{7}}$	A1	1.1b
	(5)	