

QQQ – Further Mechanics 1 – Chapter 4 – Collisions in One Dimension (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. A particle Q of mass $3m$ is at rest on a smooth horizontal plane. A particle P of mass m is moving along the plane when it collides directly with Q .

The speed of P immediately before the collision is u .

The direction of motion of P is reversed by the collision.

The coefficient of restitution between P and Q is e .

(a) Show that the speed of P immediately after the collision is $\frac{u(3e - 1)}{4}$ (6)

(b) State the full range of possible values of e . (1)

Given that $e = \frac{1}{2}$

(c) find, in terms of m and u , the magnitude of the impulse exerted by P on Q in the collision. (3)

(Total 10 marks)

2. A particle A of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal plane. Particle A collides directly with a particle B of mass m which is at rest on the plane.

The coefficient of restitution between A and B is e , where $e > 0$

(a) Show that the speed of B immediately after the collision is $2u(1 + e)$. (6)

After the collision, B hits a smooth fixed vertical wall which is perpendicular to the direction of motion of B .

(b) Show that there will be a second collision between A and B . (3)

The coefficient of restitution between B and the wall is $\frac{1}{2}$

Find, **in simplified form**, in terms of m , u and e ,

(c) the magnitude of the impulse received by B in its collision with the wall, (3)

(d) the loss in kinetic energy of B due to its collision with the wall. (3)

(Total 15 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$\begin{array}{ccc} u \rightarrow & & \rightarrow 0 \\ (P) m & & 3m (Q) \\ v \leftarrow & & \rightarrow w \end{array}$		
Use of CLM	M1	3.1a
$-mv + 3mw = mu$	A1	1.1b
Use of NEL	M1	3.4
$v + w = eu$	A1	1.1b
Solve for v :	M1	1.1b
$\frac{u(3e - 1)}{4}^*$	A1*	2.2a
	(6)	

Notes

N.B. When checking for consistency between their equations, mark the CLM equation FIRST.

M1: Use of CLM, with correct no. of terms, condone sign errors and consistent missing m 's

A1: Correct unsimplified equation. Allow v replaced by $-v$

M1: Use of NEL with e on the correct side of the equation, condone sign errors.

A1: Correct unsimplified equation consistent with CLM equation.

M1: Solve for v (must be dimensionally correct but allow slips in algebra)

A1*: Given answer correctly obtained, with no errors seen.

Allow $\frac{u}{4}(3e - 1)$ or $\frac{1}{4}u(3e - 1)$ or $\frac{u}{4}(-1 + 3e)$ or $\frac{1}{4}u(-1 + 3e)$ or $\frac{u(-1 + 3e)}{4}$

If they have v in the initial direction of P and obtain $v = \frac{u(1 - 3e)}{4}$, we need to see a clear explanation of why the signs are changed.

Mark scheme (b)

Scheme	Marks	AO
$1 \geq e > \frac{1}{3}$	B1	2.2a
	(1)	

Notes

B1: cao

Mark scheme (c)

Scheme	Marks	AO
Use of impulse-momentum for P or Q	M1	3.1a
$P: \pm m(v + u)$ OR $Q: \pm 3mw$	A1	1.1b
$\frac{9mu}{8}$ or $1\frac{1}{8}mu$	A1	1.1b
	(3)	
	(10 marks)	

Notes

M1: Condone sign errors but must have correct terms (M0 if m omitted).

M0 if m is used with w or $3m$ is used with v .

A1: $\pm m(v + u)$ or $\pm 3mw$.

N.B. v and w do not need to be substituted.

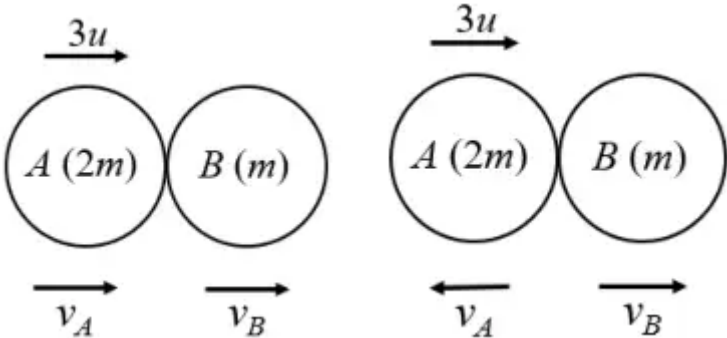
A1: Accept $1.1mu$ or better.

N.B. Must be of form kmv and must be positive.

2.

Mark scheme (a)

Scheme	Marks	AO
--------	-------	----

		
CLM:	M1	3.1a
$2m \times 3u = 2mv_A + mv_B \quad (6u = 2v_A + v_B)$ OR $2m \times 3u = -2mv_A + mv_B \quad (6u = -2v_A + v_B)$	A1	1.1b
Impact Law:	M1	3.4
$3ue = -v_A + v_B \quad \text{OR} \quad 3ue = v_A + v_B$	A1	1.1b
Solve for v_B	M1	2.1
$(v_B =) 2u(1 + e) *$	A1*	2.2a
	(6)	

Notes

M1: Use of CLM, all terms required, dimensionally correct (mass $\times$ velocity in each term). Mass and velocity paired correctly. Condone sign errors on velocities. Condone consistent extra g and/or consistent missing m (in every term).

A1: Correct unsimplified equation.

M1: Correct use of Impact Law, dimensionally correct, condone sign errors on velocity.

M0 if separation and approach are on the wrong sides.

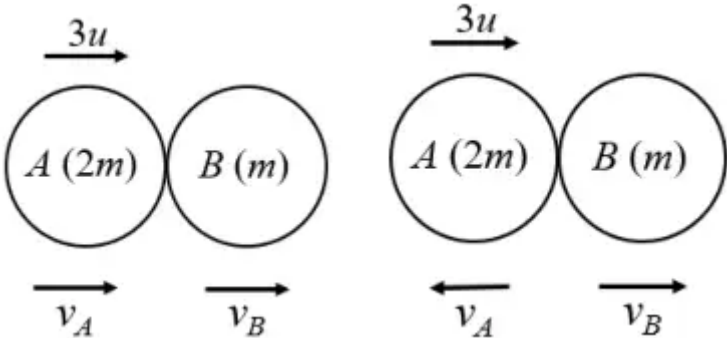
A1: Correct unsimplified equation, the direction of A must be consistent with their CLM.

M1: Use their *correctly formed* equations to solve for v_B

A1*: Given answer correctly obtained and exactly as printed. Working should include an equation in v_B only *before* reaching the given answer.

Mark scheme (b)

Scheme	Marks	AO
--------	-------	----

		
Solve for v_A	M1	3.1a
$v_A = u(2 - e)$ OR $v_A = u(e - 2)$	A1	1.1b
Complete and correct explanation: $0 \leq e \leq 1 \Rightarrow v_A > 0$ $\Rightarrow A$ continues to move towards the wall $\Rightarrow A$ will collide again with B. OR Complete and correct explanation: $0 \leq e \leq 1 \Rightarrow v_A < 0$ $\Rightarrow A$ continues to move towards the wall $\Rightarrow A$ will collide again with B.	A1	2.4
	(3)	

Notes

M1: Use given answer in (a) to solve for v_A . If seen in (a) it must be used in (b) to earn this mark.

A1: Correct expression seen for velocity of A after impact.

A1*: Correct and complete explanation with no incorrect statements. Must include **all** of:

- $0 < e \leq 1$ or $0 \leq e \leq 1$ or $0 \leq e < 1$ or 'for all e '
- $v_A > 0$ or $v_A < 0$ (must be correct for their v_A)
- Must refer to the wall.
- A continues to move with unchanged direction or towards the wall (eg do not accept descriptions for direction of travel as 'to the right' or similar)
- Conclude second collision between A and B

Note: A0* for explanations that rely on comparing speed of v_B and v_A

Mark scheme (c)

Scheme	Marks	AO
Rebound speed or velocity of $B = \pm \frac{1}{2} \times 2u(1 + e)$	B1	3.4
$\pm m [-u(1 + e) - 2u(1 + e)]$	M1	3.1a
$3(1 + e)mu$	A1	1.1b
	(3)	

Notes

B1: Correct unsimplified expression for rebound speed or velocity of B , may be seen on diagram or elsewhere in working. Accept positive or negative.

M1: Attempt to find difference in momenta, dimensionally correct (mass $\times$ velocity). Must use given answer from (a) for V_b . Condone use of e or e' in place of $\frac{1}{2}$

A1: Correct answer, must be positive. Accept any equivalent form that is either factorised or reduced to 2 terms eg $3(1 + e)mu$, $3mu + 3emu$

Mark scheme (d)

Scheme	Marks	AO
Attempt at KE loss of B	M1	3.4
$\frac{1}{2}m \left[(2u(1 + e))^2 - (u(1 + e))^2 \right]$	A1	3.1a
$\frac{3mu^2(1 + e)^2}{2}$	A1	1.1b
	(3)	
	(15 marks)	

Notes

M1: Attempt at KE loss of B : clear attempt at a difference in KE of B before and after impact with the wall. Must use velocity of B from (a). Condone use of e or e' in place of $\frac{1}{2}$. Dimensionally correct, condone subtraction either way round.

A1: Correct unsimplified expression for KE loss of B

A1: Any equivalent **factorised** form eg $\frac{3mu^2(1 + e)^2}{2}$, $\frac{3mu^2(1 + 2e + e^2)}{2}$, oe