

QQQ – Further Mechanics 1 – Chapter 3 – Elastic Strings, Springs & Hooke's Law (Version 2)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. A particle of mass 0.8 kg is attached to one end of a light elastic string of natural length 0.6 m. The other end of the string is attached to a fixed point A . The particle is released from rest at A and comes to instantaneous rest 1.1 m below A .

Find the modulus of elasticity of the string.

(4)

(Total 4 marks)

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2. A light elastic string has natural length 8 m and modulus of elasticity 80 N. The ends of the string are attached to fixed points P and Q which are on the same horizontal level and 12 m apart. A particle is attached to the mid-point of the string and hangs in equilibrium at a point 4.5 m below PQ .

(a) Calculate the weight of the particle.

(6)

(b) Calculate the elastic energy in the string when the particle is in this position.

(3)

(Total 9 marks)

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3. A spring of natural length a has one end attached to a fixed point A . The other end of the spring is attached to a package P of mass m .

The package P is held at rest at the point B , which is vertically below A such that $AB = 3a$.

After being released from rest at B , the package P first comes to instantaneous rest at A .

Air resistance is modelled as being negligible.

By modelling the spring as being light and modelling P as a particle,

(a) show that the modulus of elasticity of the spring is $2mg$ **(5)**

(b) **(i)** Show that P attains its maximum speed when the extension of the spring is $\frac{1}{2}a$

(ii) Use the principle of conservation of mechanical energy to find the maximum speed, giving your answer in terms of a and g .

(6)

In reality, the spring is not light.

(c) State one way in which this would affect your energy equation in part (b). **(1)**

(Total 12 marks)

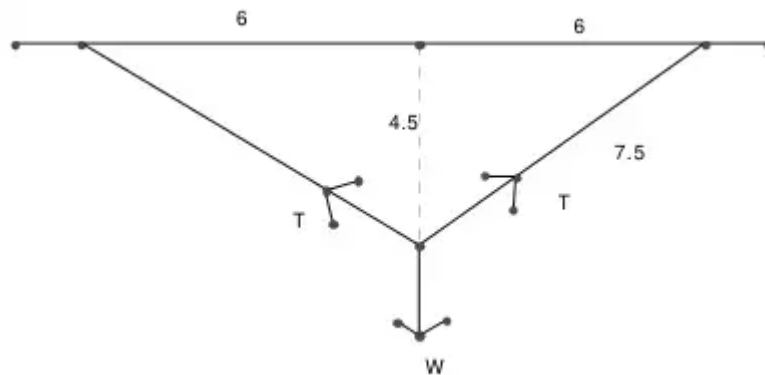
Solutions

1.

Scheme	Marks
$\text{EPE} = \frac{\lambda \times 0.5^2}{1.2}$	B1
GPE lost = EPE gained	M1 (used)
$0.8 \times 9.8 \times 1.1 = \frac{\lambda \times 0.5^2}{1.2}$	A1ft
$\lambda = 41.4 \text{ N or } 41 \text{ N}$	A1
	(4 marks)

2.

Mark scheme (a)



Scheme	Marks
Resolving vertically: $2T \cos \theta = W$	M1A2,1,0
Hooke's Law: $T = \frac{80 \times 3.5}{4}$	M1A1
$W = 84\text{N}$	A1

Mark scheme (b)

Scheme	Marks
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$\text{EPE} = 2 \times \frac{80 \times 3.5^2}{2 \times 4}, = 245 \text{ (or awrt 245)}$ (alternative $\frac{80 \times 7^2}{16} = 245$)	M1A1ft,A1
	(9 marks)

3.

Mark scheme (a)

Scheme	Marks	AO
EPE at $A = \frac{\lambda a^2}{2a}$ or EPE at $B = \frac{\lambda(2a)^2}{2a}$	M1	2.1
Form work-energy equation:	M1	3.3
$\frac{\lambda a^2}{2a} + mg \times 3a = \frac{\lambda(2a)^2}{2a} \quad \left(\frac{\lambda a}{2} + 3mga = 2\lambda a \right)$	A1 A1	1.1b 1.1b
$3mg = \frac{3\lambda}{2} \Rightarrow \lambda = 2mg \quad *$	A1*	2.2a
	(5)	

Notes

M1: Correct method for EPE seen or implied

Need something of the form $\frac{1}{2}kx^2$ where $k = \frac{\lambda}{a}$

Must be using the formula for EPE correctly at least once

M1: Require all terms. Dimensionally correct. Condone their EPE. Condone sign errors

A1 A1: Unsimplified equation with at most one error. A repeated error in EPE formula is one error

Correct unsimplified equation.

A1*: Obtain **given answer** from correct working

Mark scheme (b)

Scheme	Marks	AO
Extension at equilibrium:	M1	2.1
$\frac{2mgx}{a} = mg \Rightarrow x = \frac{a}{2} \quad *$	A1*	1.1b
Use work-energy equation to find max speed:	M1	3.4

$\frac{2mgx^2}{2a} + mg \times (2a - x) + \frac{1}{2}mV^2 = \frac{2mg(2a)^2}{2a}$ $\left(\frac{ag}{4} + \frac{3ag}{2} + \frac{1}{2}V^2 = 4ag \right)$	A1 A1	1.1b 1.1b
$V = 3\sqrt{\frac{ag}{2}}$	A1	2.2a
	(6)	

Notes

M1: Use correct method for tension to find the extension at equilibrium. Need to see the formula for tension **used**.

Allow verification with an appropriate conclusion

If they use SHM they must use $F = ma$ to prove that P is moving with SHM, otherwise 0/2.

A1*: Correct answer from correct work

Allow verification **with an appropriate conclusion**

M1: Use given x to form work-energy equation. Need all terms, and dimensionally correct. Condone sign errors.

Accept with values of λ and x not substituted

A1 A1: Unsimplified equation with at most one error. Need given λ and given x substituted at some point.

A repeated error in the formula for EPE is one error.

Correct unsimplified equation with given λ and given x substituted at some point

A1: Use correct method for tension to find the extension at equilibrium. Any equivalent form. $2.1\sqrt{ag}$ or better

Alternative for the first M1A1

Scheme	Marks	AO
Use the work-energy equation to obtain $\frac{dV^2}{dx}$ and set the derivative equal to zero	M1	
$\frac{1}{a} \times 2x - 1 = 0 \Rightarrow x = \frac{a}{2}$	A1	

Alt: M1: Or an equivalent method for finding the turning point of a quadratic

Alt: A1*: Correct answer from correct work

Mark scheme (c)

Scheme	Marks	AO
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e.g. for B1 Need to include the GPE of the spring The extension of the spring at equilibrium will be different The spring will have KE You would need to include the KE of the spring in the energy equation You would need to include the GPE of the spring in the energy equation The GPE of the system changes It would take work to raise the spring so the package would have less KE If the spring has mass then GPE of the spring would need to be included	B1	3.5b
	(1)	
	(12 marks)	

Notes

B1: Any valid response.

B0 if answer includes an additional incorrect factor. Must be specific e.g. not just “the GPE changes”, but the GPE of the system changes is OK.

Must relate to an effect on the energy equation

E.g. for B0

The extension changes

AB will increase

The tension/energy/GPE/work done etc would increase

The KE/GPE/EPE/acceleration/extension/velocity changes

The mass of the spring would drag down and the EPE would change

The EPE/KE/GPE etc would be variable

There would be tension in the spring as well

It has weight

The velocity would decrease as energy is converted