

QQQ – Decision 2 – Chapter 7 – Recurrence Relations (Version 2)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1. The general solution of the second order recurrence relation

$$u_{n+2} + k_1 u_{n+1} + k_2 u_n = 0 \quad n \geq 0$$

is given by

$$u_n = (A + Bn)(-3)^n$$

where A and B are arbitrary non-zero constants.

(a) Find the value of k_1 and the value of k_2 (2)

Given that $u_0 = u_1 = 1$

(b) find the value of A and the value of B . (2)

(Total 4 marks)

Edexcel A2 June 2022 Q2

2. The complementary function for the second order recurrence relation

$$u_{n+2} + \alpha u_{n+1} + \beta u_n = 20(-3)^n \quad n \geq 0$$

is given by

$$u_n = A(2)^n + B(-1)^n$$

where A and B are arbitrary non-zero constants.

(a) Find the value of α and the value of β . (2)

Given that $2u_0 = u_1$ and $u_4 = 164$

(b) find the solution of this second order recurrence relation to obtain an expression for u_n in terms of n . (6)

(Total 8 marks)

Edexcel A2 October 2020 Q4

3. Peter sets up a savings plan. He makes an initial deposit of $\pounds D$ and then pays in $\pounds M$ at the end of each month.

The value of the savings plan, in pounds, is modelled by

$$u_{n+1} = 1.025 u_n + 1800$$

where $n \geq 0$ is an integer and u_n is the total value of the savings plan, in pounds, after n years.

(a) Calculate the value of M **(1)**

Given that the value of the savings plan after 1 year is $\pounds 6925$

(b) solve the recurrence relation for u_n **(5)**

(c) Determine the value of D **(1)**

(d) Hence determine, using algebra, the number of years it will take for the value of the savings plan to exceed $\pounds 20000$ **(3)**

(Total 10 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$(m + 3)^2 = 0$	M1	3.1a
$k_1 = 6$ and $k_2 = 9$	A1	1.1b
	(2)	

Notes

M1: Correct auxiliary equation (may be implied by 6, 9 correct) from $u_n = (A + Bn)(-3)^n$

A1: CAO – allow values stated implicitly i.e., $u_{n+2} + 6u_{n+1} + 9u_n = 0$

Mark scheme (b)

Scheme	Marks	AO
$u_0 = 1 \Rightarrow A(-3)^0 = 1$ $u_1 = 1 \Rightarrow (A + B)(-3) = 1$	M1	1.1b
$A = 1$ and $B = -\frac{4}{3}$	A1	1.1b
	(2)	
	(4 marks)	

Notes

M1: Uses $u_0 = u_1 = 1$ and attempts to find A and B

A1: CAO – allow values stated implicitly e.g. $u_n = \left(1 - \frac{4}{3}n\right)(-3)^n$

2.

Mark scheme (a)

Scheme	Marks	AO
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CF of $u_n = A(2)^n + B(-1)^n \Rightarrow$ auxiliary equation is $(m - 2)(m + 1) = 0$	M1	3.1a
$m^2 - m - 2 = 0 \Rightarrow \alpha = -1, \beta = -2$	A1	2.2a
	(2)	

Notes

M1: Uses given complementary function to find auxiliary equation corresponding to second-order recurrence relation

A1: cao for both α and β

Mark scheme (b)

Scheme	Marks	AO
particular solution try $u_n = \lambda(-3)^n, u_{n+1} = \lambda(-3)^{n+1}, u_{n+2} = \lambda(-3)^{n+2}$	M1	2.1
$9\lambda + 3\lambda - 2\lambda = 20 (\Rightarrow \lambda = 2)$	M1	1.1b
$u_n = A(2)^n + B(-1)^n + 2(-3)^n$	A1	1.1b
$2u_0 = u_1 \Rightarrow 2A + 2B + 4 = 2A - B - 6$	M1	1.1b
$u_4 = 164 \Rightarrow 16A + B + 162 = 164$	M1	1.1b
$A = \frac{1}{3}, B = -\frac{10}{3} \Rightarrow u_n = \frac{1}{3}(2)^n - \frac{10}{3}(-1)^n + 2(-3)^n$	A1	2.2a
	(6)	
	(8 marks)	

Notes

M1: substitute $u_n = \lambda(-3)^n$ into their second-order recurrence relation

M1: forms linear equation in λ only

A1: correct general solution

M1: use $2u_0 = u_1$ to form an equation in B (and possibly A)

M1: use $u_4 = 164$ to set up a second equation in A and B

A1: cao

3.

Mark scheme (a)

Scheme	Marks	AO
$M = 150$	B1	3.4
	(1)	

Notes

B1: CAO

Mark scheme (b)

Scheme	Marks	AO
(aux equation $m - 1.025 = 0 \Rightarrow$) complementary function is $A(1.025)^n$	B1	1.1b
Consider a trial solution of the form $u_n = \lambda$ so $\lambda - 1.025\lambda = 1800 \Rightarrow \lambda = \dots$	M1	1.1b
General solution is $u_n = A(1.025)^n - 72000$	A1	1.1b
$n = 1, u_1 = 6925 \Rightarrow A = \dots$	M1	3.4
$u_n = 77000(1.025)^n - 72000$	A1	1.1b
	(5)	

Notes

B1: CAO

M1: substituting their trial solution into the recurrence relation in an attempt to find their λ

A1: CAO

M1: using the conditions in the model to calculate A

A1: CAO (must be $u_n =$ not $u_{n+1} =$)

Alternative

General solution is $u_n = A(1.025)^{n-1} - 72000$

$u_n = 78925(1.025)^{n-1} - 72000$

Mark scheme (c)

Scheme	Marks	AO
$D = 77000 - 72000 \Rightarrow D = \text{£}5000$	B1ft	1.1b
	(1)	

Notes

B1ft: substitutes $n = 0$ in their solution to find D

Alternative

$$6925 = 1.025D + 1800 \Rightarrow D = 5000$$

Mark scheme (d)

Scheme	Marks	AO
$77000(1.025)^n - 72000 > 20000$	M1	1.1b
$(1.025)^n > \frac{92}{77} \Rightarrow n \log(1.025) > \log\left(\frac{92}{77}\right)$	M1	1.1b
$n > 7.20795 \dots \Rightarrow n = 8$	A1	1.1b
	(3)	
	(10 marks)	

Notes

M1: sets their particular solution greater than 20000– their particular solution must be of the correct form ($u_n = c(1.025)^n \pm d$ or $u_n = c(1.025)^{n-1} \pm d$)

M1: dependent on previous M mark – correctly re-arranges and then applies the process of taking logs for their particular solution

A1: CAO

Alternative

$$78925(1.025)^{n-1} - 72000 > 20000$$

$$(1.025)^{n-1} > \frac{3680}{3157} \Rightarrow (n-1) \log(1.025) > \log\left(\frac{3680}{3157}\right)$$

$$n-1 > 6.20795 \dots \Rightarrow n = 8$$