

QQQ – Decision 2 – Chapter 7 – Recurrence Relations (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1.

(a) Find the general solution of the recurrence relation

$$u_{n+1} = 3u_n + 2^n \quad n \geq 1$$

(4)

(b) Find the particular solution of this recurrence relation for which $u_1 = u_2$

(2)

(Total 6 marks)

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2. Martina decides to open a bank account to help her to save for a holiday. Each month she puts £ k into the account and allows herself to spend one quarter of what was in the account at the end of the previous month.

Let u_n (where $n \geq 1$) represent the amount in the account at the end of month n .

Martina has £ k in the account at the end of the first month.

(a) By setting up a first order recurrence relation for u_{n+1} in terms of u_n , determine an expression for u_n in terms of n and k .

(6)

At the end of the 8th month, Martina needs to have at least £1750 in the account to pay for her holiday.

(b) Determine, to the nearest penny, the minimum amount of money that Martina should put into the account each month.

(2)

(Total 8 marks)

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3. An increasing sequence $\{u_n\}$ for $n \in \mathbb{N}$ is such that the difference between the n th term of $\{u_n\}$ and the mean of the previous two terms of $\{u_n\}$ is always 6

(a) Show that, for $n \geq 3$

$$2u_n - u_{n-1} - u_{n-2} = 12$$

(2)

Given that $u_1 = 2$ and $u_2 = 8$

(b) find the solution of this second order recurrence relation to obtain an expression for u_n in terms of n . **(7)**

(c) Show that as $n \rightarrow \infty$, $u_n \rightarrow kn$ where k is a constant to be determined. You must give reasons for your answer. **(2)**

(Total 11 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
$u_{n+1} = 3u_n + 2^n \quad n \geq 1$		
(aux equation $m - 3 = 0 \Rightarrow$) complementary function is $A(3)^n$	B1	1.1b
Consider a trial solution of the form $u_n = k(2^n)$ so $2k(2^n) = 3k(2^n) + 2^n$	M1	1.1b
$k = -1$	A1	1.1b
General solution is $u_n = A(3)^n - 2^n$	A1	1.1b
	(4)	

Notes

B1: CAO for complementary function

M1: substituting correct trial solution into the recurrence relation – allow substitution of $u_n = k(2^n)$ into $u_n = 3u_{n-1} + 2^{n-1}$ but not $u_n = 3u_{n-1} + 2^n$

A1: CAO $k = -1$

A1: CAO for the general solution – must include $u_n = \dots$

Mark scheme (b)

Scheme	Marks	AO
$u_1 = u_2 \Rightarrow 3A - 2 = 9A - 4 \Rightarrow A = \dots$	M1	3.1a
$u_n = 3^{n-1} - 2^n$	A1	1.1b
	(2)	
	(6 marks)	

Notes

M1: using the condition $u_1 = u_2$ to calculate a value for their $A (= \frac{1}{3})$ - this mark is dependent on the general solution being of the form $\pm \lambda(3)^n \pm \mu(2)^n$

A1: CAO for the particular solution (oe) – must include $u_n = \dots$ - however, if neither (general nor particular) solution is given in terms of u_n then award this mark if correct expression in terms of n seen (or if both solutions are given in terms of say u_{n+1})

2.

Mark scheme (a)

Scheme	Marks	AO
$u_{n+1} = 0.75u_n + k$	B1	3.3
(aux equation $m - 0.75 = 0 \Rightarrow$) complementary function is $A(0.75)^n$	B1	1.1b
Consider a trial solution of the form $u_n = \lambda$ so $\lambda - 0.75\lambda = k$	M1	1.1b
General solution is $u_n = A(0.75)^n + 4k$	A1	1.1b
$u_1 = k \therefore 0.75A + 4k = k \Rightarrow A = \dots$	M1	3.4
$u_n = 4k(1 - (0.75)^n)$	A1	2.2a
	(6)	

Notes

B1: CAO

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M1: substituting their trial solution into the recurrence relation (which must be of the form $u_{n+1} = \alpha u_n + k$ where $\alpha = 0.75$ or ± 0.25) in an attempt to find their λ

A1: CAO for the general solution (may be implied by subsequent working)

M1: using the conditions in the model and their expression of the form $A(\alpha)^n + Bk$ where α is 0.75 or ± 0.25 to calculate A (which if correct is $-4k$)

A1: CAO for the particular solution (may be an expression without $u_n =$ accept any equivalent form e.g. $u_n = 4k - 3k(0.75)^{n-1}$)

Mark scheme (b)

Scheme	Marks	AO
$4k(1 - 0.75^8) = 1750 \Rightarrow k = \dots$	dM1	3.4

$k = 486.1721068 \dots \Rightarrow k = 486.18$	A1	2.2a
	(2)	
	(8 marks)	

Notes

dM1: Applying $n = 8$ to their general solution of the form $A(\alpha)^n + Bk$ where α is 0.75 or ± 0.25 equal to 1750 (dependent on both M marks in (a))

A1: CAO – must be 486.18

3.

Mark scheme (a)

Scheme	Marks	AO
$u_n - \frac{1}{2}(u_{n-1} + u_{n-2}) = 6 \Rightarrow 2u_n - (u_{n-1} + u_{n-2}) = 12$	1M1	2.1
$2u_n - u_{n-1} - u_{n-2} = 12$	1A1	2.2a
	(2)	

Notes

1M1: attempt to write given information in terms of a recurrence equation (allow sign errors and errors in notation)

1A1*: CAO (NB equation is provided in the question so must see a correct unsimplified recurrence relation which is simplified with no errors to the required form for this mark).

Mark scheme (b)

Scheme	Marks	AO
aux equation $2m^2 - m - 1 = 0 \Rightarrow m = 1, m = -\frac{1}{2}$	1B1	2.1
$u_n = A + B\left(-\frac{1}{2}\right)^n$	2B1	1.1b
particular solution try $u_n = \lambda n$ $\therefore 2\lambda n - \lambda n + \lambda - \lambda n + 2\lambda = 12 \Rightarrow \lambda [= 4]$	1M1	1.1b
$u_n = A + B\left(-\frac{1}{2}\right)^n + 4n$	1A1	1.1b
$u_1 = 2 \Rightarrow 2A - B = -4$	2M1	1.1b

$u_2 = 8 \Rightarrow 4A + B = 0$	3M1	1.1b
$A = -\frac{2}{3}, B = \frac{8}{3} \Rightarrow u_n = -\frac{2}{3} + \frac{8}{3} \left(-\frac{1}{2}\right)^n + 4n$	2A1	2.2a
	(7)	

Notes

1B1: CAO for auxiliary equation and corresponding solutions (this mark can be implied by correct complementary function)

2B1: complementary function CAO. Condone lack of ' $u_n =$ '.

1M1: substitutes $u_n = \lambda n$ into their 2nd-order recurrence relation and solves to obtain $\lambda = \dots$

Note: $2\lambda n - \lambda(n-1) - \lambda(n-2) = 12$ o.e. $\Rightarrow \lambda = \dots$ can earn this mark

Note: Do not condone sign errors or errors in coefficients e.g.

$2\lambda n - \lambda(n+1) - \lambda(n-2) = 12$ is M0

$2\lambda n - 2\lambda(n-1) - \lambda(n-2) = 12$ is M0

1A1: Correct general solution. Condone lack of ' $u_n =$ '.

2M1: Forms one equation in A and B . General Solution must be of the form.

$u_n = A + B \left(-\frac{1}{2}\right)^n + \mu n$ where $\mu \neq 0$

3dM1: Forms a second equation in A and B . Dependent on the previous method mark.

2A1: Particular solution CAO. Do not condone lack of ' $u_n = \dots$ '

Mark scheme (c)

Scheme	Marks	AO
$u_n \rightarrow 4n \quad (k=4)$	1M1	2.1
As $n \rightarrow \infty$, $\left(-\frac{1}{2}\right)^n \rightarrow 0$ and $4n$ is considerably greater than $-\frac{2}{3}$	1A1ft	1.1b
	(2)	
	(11 marks)	

Notes

M1: Obtains correct limit (ft their particular solution which must be of the correct form i.e.

$u_n = A + B \left(-\frac{1}{2}\right)^n + \mu n$ where $\mu \neq 0$)

A1ft: Provides correct reasoning including comments relating to both of the following:

- $n \rightarrow \infty, \left(-\frac{1}{2}\right)^n \rightarrow 0$

- $-\frac{2}{3}$ becomes negligible (compared to $4n$ as $n \rightarrow \infty$)

Note: Condone $-\frac{2}{3}$ becomes insignificant'