

QQQ – Decision 2 – Chapter 5 – Dynamic Programming (Version 1)

Total Marks: 26

(26 = Platinum, 23 = Gold, 21 = Silver, 18 = Bronze)

- 1.
- (a) State Bellman's principle of optimality. (1)
 - (b) Explain what is meant by a minimax route. (1)
 - (c) Describe a practical problem that would require a minimax route as its solution. (2)

(Total 4 marks)

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2.

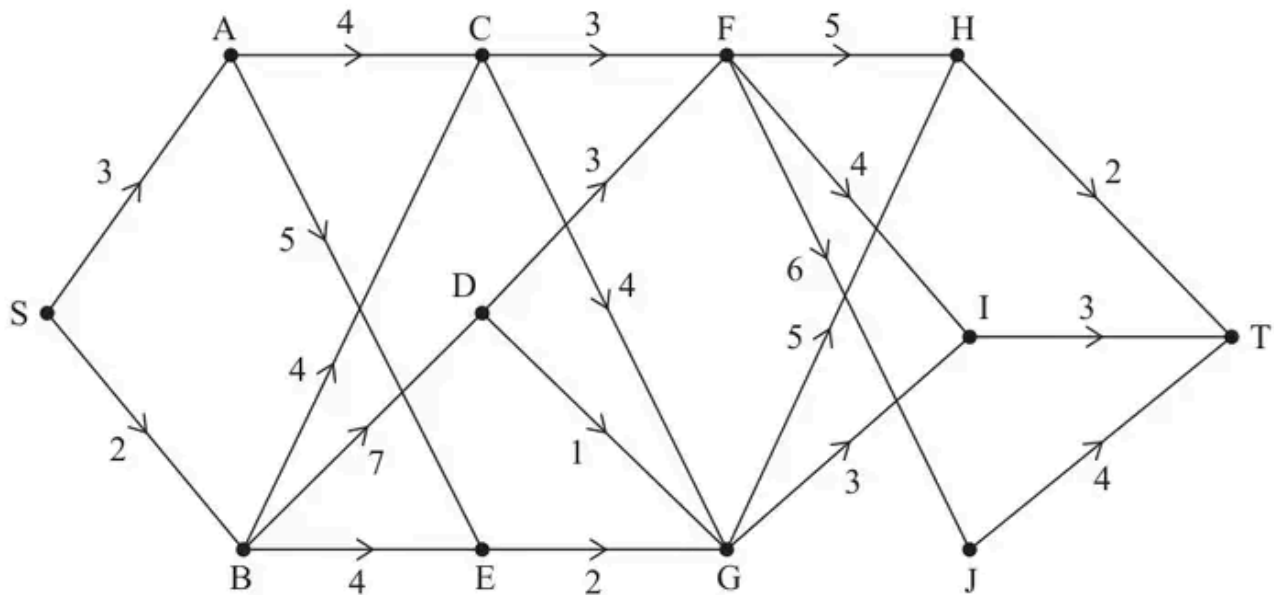


Figure 2

The staged, directed network in Figure 2 represents the roads that connect 12 towns, S, A, B, C, D, E, F, G, H, I, J and T. The number on each arc shows the time, in hours, it takes to drive between these towns.

Elena plans to drive from S to T. She must arrive at T by 9 pm.

- (a) By completing the table in the answer book, use dynamic programming to find the latest time that Elena can start her journey from S to arrive at T by 9 pm. (9)

[The table in the answer book has columns headed Stage, State, Action, Destination and Value.]

- (b) Hence write down the route that Elena should take. (1)

(Total 10 marks)

3. A manufacturer can export five batches of footwear each year. Each exported batch contains just one type of footwear. The types of footwear are trainers, sandals or high heels.

The table below shows the profit, in £1000s, for the number of batches of each type of footwear.

Number of batches	0	1	2	3	4	5
Trainers	0	50	90	170	225	295
Sandals	0	70	110	165	245	300
High heels	0	75	115	x	235	305

The total annual profit is to be maximised.

- (a)** Use dynamic programming to determine the two possible values of the maximum total annual profit, giving one of these values in terms of x . **(10)**

Given that the maximum total annual profit is £320 000

- (b)** advise the manufacturer on the possible ways in which the five batches should be allocated. **(2)**

(Total 12 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks
Any part of an optimal path is itself optimal	B1
	(1)

Notes

B1 cao (“port”, “section”, OK; “arc”, “stage”, activity”, “event”, not)

Mark scheme (b)

Scheme	Marks
The route chosen such that the maximum arc length is as small as possible	B1
	(1)

Notes

B1 cao (not min of max rate, not minimize largest arc)

Mark scheme (c)

Scheme	Marks
e.g. Maximising freight by minimising fuel needed when planning multiple stage light aircraft journey	B2, 1, 0
	(2)
	(4 marks)

Notes

B2 cao

B1 cloze “Bod” gets B1

2.

Mark scheme (a)

Scheme	Marks	AO																																			
<table><tr><th>Stage</th><th>State</th><th>Action</th><th>Dest</th><th>Value</th></tr><tr><td>1</td><td>H</td><td>HT</td><td>T</td><td>2*</td></tr><tr><td></td><td>I</td><td>IT</td><td>T</td><td>3*</td></tr><tr><td></td><td>J</td><td>JT</td><td>T</td><td>4*</td></tr></table>	Stage	State	Action	Dest	Value	1	H	HT	T	2*		I	IT	T	3*		J	JT	T	4*	B1	3.1a															
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		BE	E	4 + 8 = 12*																																	
5	S	SA	A	3 + 13 = 16																																	

		SB	B	$2 + 12 = 14^*$		
Latest time that Elena can start her journey is 7 am					A1	3.2a
					(9)	

Notes

All M marks – must bring earlier optimal results into calculations. Ignore extra rows. Penalise lack of * only once per question.

B1: CAO for first stage

M1: Second stage completed. At least 4 rows, something in each cell.

A1: CAO for second stage exactly 5 rows

M1: Third stage completed. At least 4 rows, something in each cell.

A1: CAO for third stage exactly 5 rows

M1: Fourth stage completed. 5 rows, something in each cell.

A1ft: correct ft their optimal values from third stage

A1: CAO for fifth stage

A1: Correct latest start time in context (e.g. 7 am, 07:00, etc.)

Special Case – Maximin or Minimax

Stage	State	Action	Dest	Value	Value
1	H	HT	T	2^*	2^*
	I	IT	T	3^*	3^*
	J	JT	T	4^*	4^*
2	F	FH	H	$\text{Min } (5, 2) = 2$	$\text{Max } (5, 2) = 5$
		FI	I	$\text{Min } (4, 3) = 3$	$\text{Max } (4, 3) = 4^*$
		FJ	J	$\text{Min } (6, 4) = 4^*$	$\text{Max } (6, 4) = 6$
	G	GH	H	$\text{Min } (5, 2) = 2$	$\text{Max } (5, 2) = 5^*$
		GI	I	$\text{Min } (3, 3) = 3^*$	$\text{Max } (3, 3) = 3$
3	C	CF	F	$\text{Min } (3, 4) = 3^*$	$\text{Max } (3, 4) = 4^*$
		CG	G	$\text{Min } (4, 3) = 3^*$	$\text{Max } (4, 5) = 5$
	D	DF	F	$\text{Min } (3, 4) = 3^*$	$\text{Max } (3, 4) = 4^*$
		DG	G	$\text{Min } (1, 3) = 1$	$\text{Max } (1, 5) = 5$

	E	EG	G	Min (2, 3) = 2*	Max (2, 5) = 5*
4	A	AC	C	Min (4, 3) = 3*	Max (4, 4) = 4*
		AE	E	Min (5, 2) = 2	Max (5, 5) = 5
	B	BC	C	Min (4, 3) = 3*	Max (4, 4) = 4*
		BD	D	Min (7, 3) = 3*	Max (7, 4) = 7
		BE	E	Min (4, 2) = 2	Max (4, 5) = 5
5	S	SA	A	Min (3, 3) = 3*	Max (3, 4) = 4*
		SB	B	Min (2, 3) = 2	Max (2, 4) = 4*

B1 M1 A0 M1 A0 M1 A0 A0 A0 B0 - Max 4/10

Mark scheme (b)

Scheme	Marks	A0
Route: SBEGIT	B1	2.2a
	(1)	
	(10 marks)	

Notes

B1: Correct route (dependent on all previous M marks)

3.

Mark scheme (a)

Scheme					Marks	AO																																			
<table><tr><th>Stage</th><th>State</th><th>Action</th><th>Dest.</th><th>Value</th></tr><tr><td>Trainers</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td></td><td>1</td><td>1</td><td>0</td><td>50</td></tr><tr><td></td><td>2</td><td>2</td><td>0</td><td>90</td></tr><tr><td></td><td>3</td><td>3</td><td>0</td><td>170</td></tr><tr><td></td><td>4</td><td>4</td><td>0</td><td>225</td></tr><tr><td></td><td>5</td><td>5</td><td>0</td><td>295</td></tr></table>					Stage	State	Action	Dest.	Value	Trainers	0	0	0	0		1	1	0	50		2	2	0	90		3	3	0	170		4	4	0	225		5	5	0	295	B1	3.1a
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						4	4	0	225																																
	5	5	0	295																																					

Stage	State	Action	Dest.	Value
Sandals	0	0	0	0
	1	1	0	$70 + 0 = 70^*$
		0	1	$0 + 50 = 50$
	2	2	0	$110 + 0 = 110$
		1	1	$70 + 50 = 120^*$
		0	2	$0 + 90 = 90$
	3	3	0	$165 + 0 = 165$
		2	1	$110 + 50 = 160$
		1	2	$70 + 90 = 160$
		0	3	$0 + 170 = 170^*$

M1

A1

A1

3.1a

1.1b

1.1b

Stage	State	Action	Dest.	Value
	4	4	0	$245 + 0 = 245^*$
		3	1	$165 + 50 = 215$
		2	2	$110 + 90 = 200$
		1	3	$70 + 170 = 240$
		0	4	$0 + 225 = 225$
	5	5	0	$300 + 0 = 300^*$
		4	1	$245 + 50 = 295$
		3	2	$165 + 90 = 255$
		2	3	$110 + 170 = 280$
		1	4	$70 + 225 = 295$
		0	5	$0 + 295 = 295$

M1

A1

A1

1.1b

1.1b

1.1b

Stage	State	Action	Dest.	Value
High heels	5	5	0	$305 + 0 = 305$

M1

A1ft

1.1b

1.1b

		4	1	$235 + 70 = 305$		
		3	2	$x + 120$		
		2	3	$115 + 170 = 285$		
		1	4	$75 + 245 = 320$		
		0	5	$0 + 300 = 300$		
320 and $x + 120$					A1ft	1.1b
					(10)	

Notes

B1: CAO for the first stage (all six rows) – entries in all columns must be correct – candidates may start with state 5 (rather than state 0) which is fine

M1: Second stage – my states 1, 2 and 3 (so at least 9 rows in the first half of the second stage or at least 20 non-zero rows). Value column must be complete with at least one value correct for each state – ignore entries in all other columns

A1: Value column for states 1, 2 and 3 correct for the second stage – ignore entries in all other columns and condone additional rows

A1: CAO for states 0, 1, 2 and 3 of the second stage (no additional rows for these four states) – entries in all columns must be correct

M1: Second stage – my states 4 and 5 (so at least 11 rows in the second half of the second stage or at least 20 non-zero rows). Value column must be complete with at least one value correct for each state – ignore entries in all other columns

A1: Value column for states 4 and 5 correct for the second stage – ignore entries in all other columns and condone additional rows

A1: CAO for states 4 and 5 of the second stage (no additional rows for these two states) - entries in all columns must be correct

M1: At least 6 rows for the third stage. Value column must be complete with at least 3 values correct – ignore entries in all other columns

A1ft: CAO for third stage correct (no additional rows for this stage) - entries in all columns must be correct

A1ft: Must have earned all previous M marks from their completed dynamic programming but ft their result

Mark scheme (b)

Scheme	Marks	AO
Trainers: 0 Sandals: 4 High heels: 1	B1	1.1b
Trainers: 1 Sandals: 1 High heels: 3	B1	2.2a
	(2)	
	(12 marks)	

Notes

B1: One correct allocation (dependent on first three M marks in (a))

B1: For both correct (dependent on first three M marks in (a))