

QQQ – Decision 2 – Chapter 4 – Flows in Networks 2 (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1.

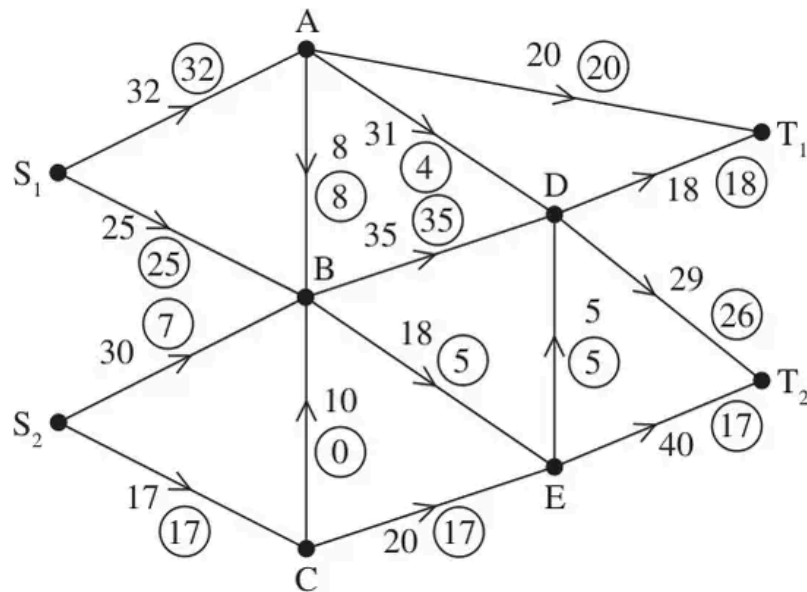


Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

- (a) (i) Add a supersource, S, and a supersink, T, and corresponding arcs to Diagrams 1 and 2, in the answer book. **(3)**
- (ii) Enter the flow value and appropriate capacity on each of the arcs you have added to Diagram 1.
- (b) Complete the initialisation of the labelling procedure on Diagram 2 by entering values along the new arcs from S and T, and along AB, AD and DT₂. **(2)**
- (c) Hence use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow. **(4)**
- (d) Draw a maximal flow pattern on Diagram 3 in the answer book. **(2)**
- (e) Prove that your flow is maximal. **(2)**

(Total 13 marks)

2.

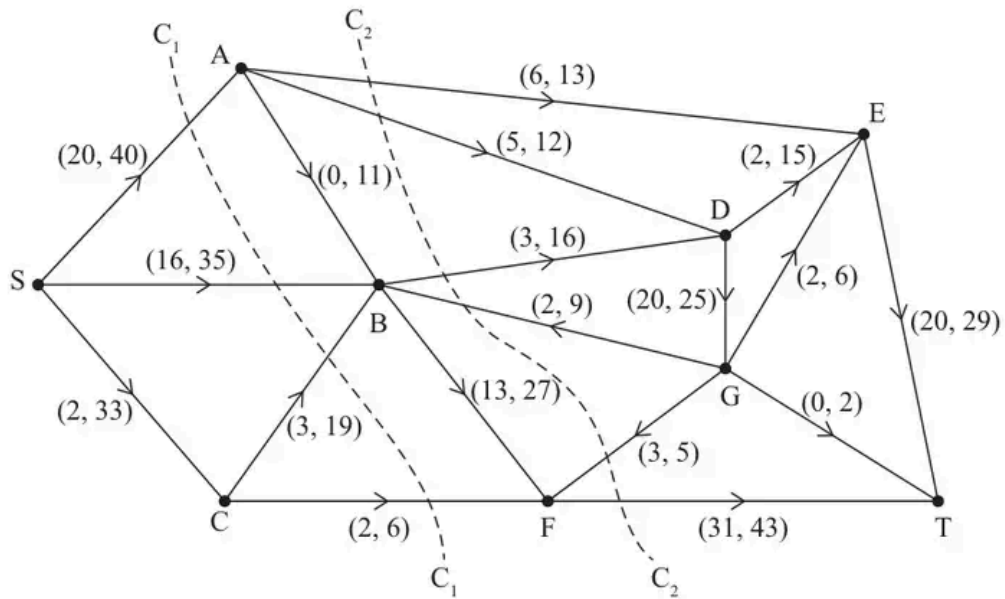


Figure 2

Figure 2 shows a capacitated, directed network. The network represents a system of pipes through which fluid can flow.

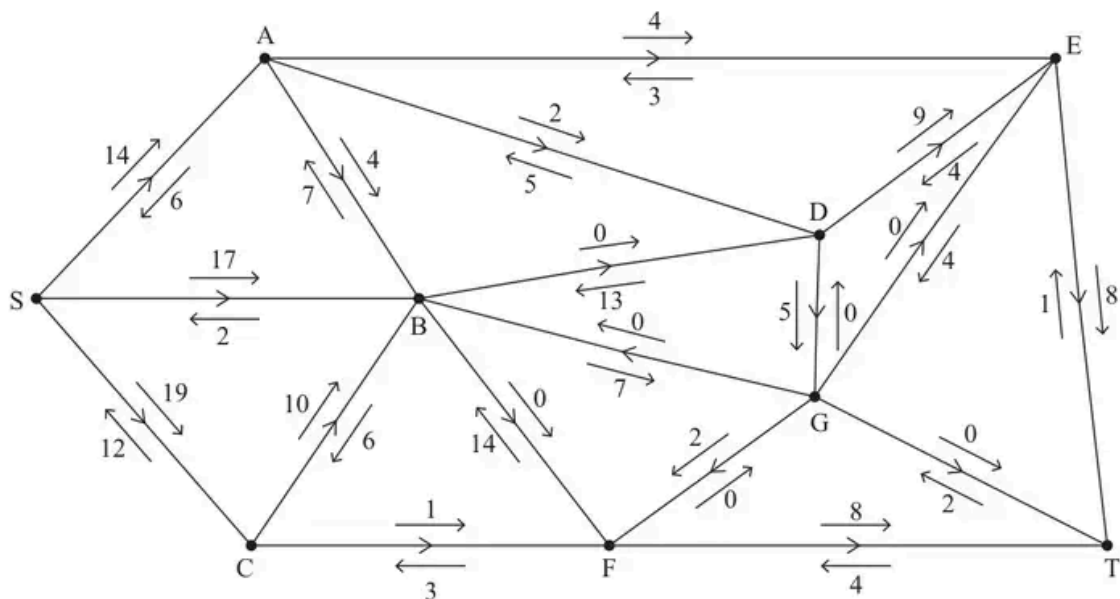
The weights on the arcs show the lower and upper capacities for the corresponding pipes, in litres per second.

(a) Calculate the capacity of

(i) cut C_1

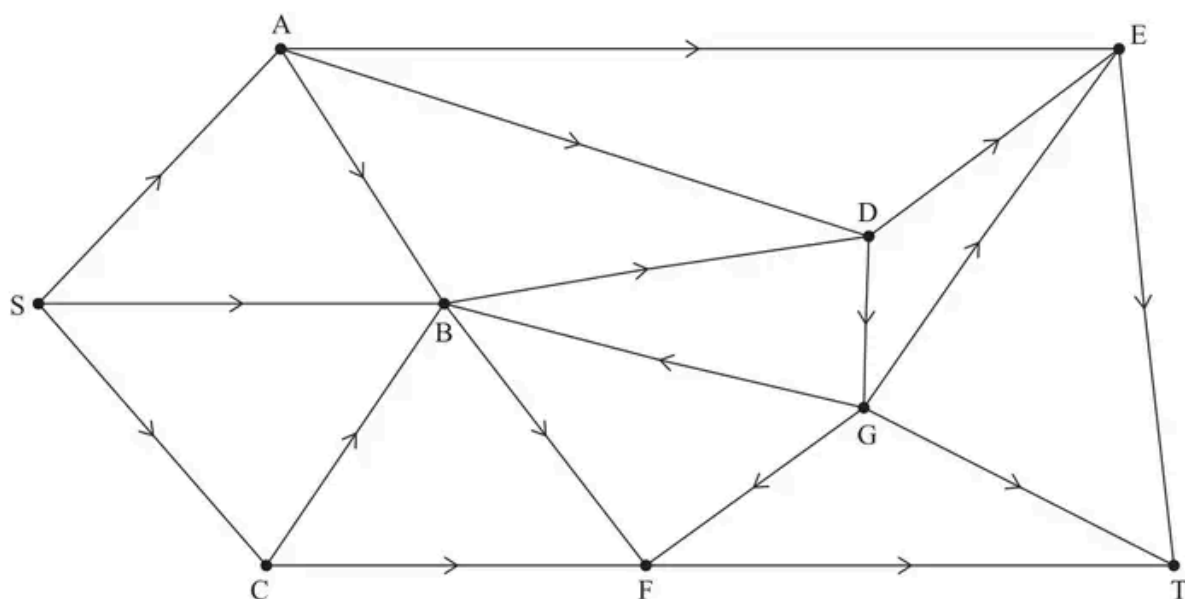
(ii) cut C_2 (2)

(b) Using only the capacities of cuts C_1 and C_2 , state what can be deduced about the maximum flow through the system. (1)



A scientist monitors the system of pipes in which a fluid now flows from the source, S, to the sink, T.

The scientist initialises the labelling procedure for this system. The excess and potential backflows for this are shown on the arrows either side of each arc in Figure 3.



(f) Prove that the answer to part (e) is optimal.

(3)

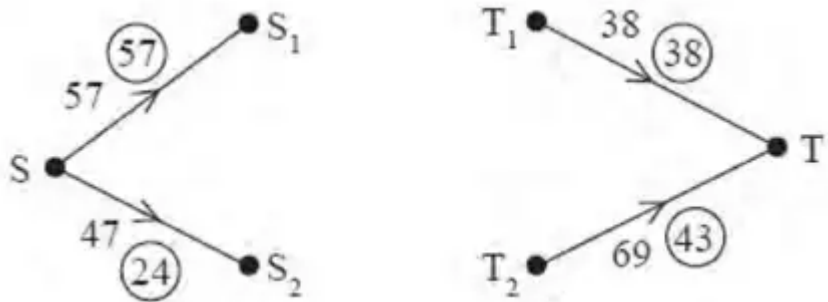
(Total 11 marks)

Edexcel A2 June 2025 Q4

Solutions

1.

Mark scheme (a)

Scheme	Marks
	M1 A1 A1
	(3)

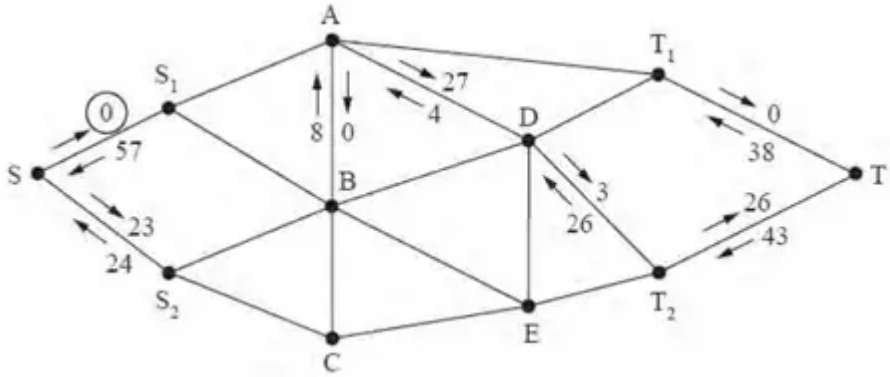
Notes

a1M1: Four arcs added, SS_1 , SS_2 , T_1T , T_2T and 2 numbers on each.

a1A1: CAO for arcs

a2A1: CAO for flow values and capacities

Mark scheme (b)

Scheme	Marks
	M1 A1
	(2)

Notes

b1M1: Two numbers on each arc and at least three arcs or six numbers correct.

b1A1: CAO do give bod since they might well cross these numbers out.

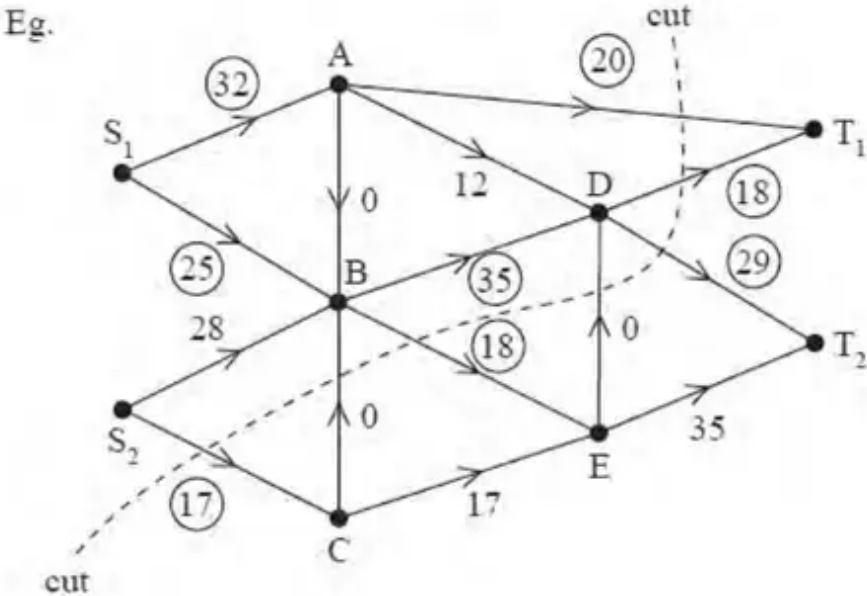
Mark scheme (c)

Scheme	Marks
E.g. SS ₂ BET ₂ T – 13 SS ₂ BADT ₂ T – 3 SS ₂ BADET ₂ T – 5	M1 A1 A1 A1
	(4)

Notes

- c1M1:** One valid flow augmenting route found and a value stated.
- c1A1:** Flow increased by at least 3.
- c2A1:** A second correct flow route of value at least 5 and value correct.
- c3A1:** CSO Flow increased by 21 and no more.

Mark scheme (d)

Scheme	Marks
E.g. 	M1 A1
	(2)

Notes

- d1M1:** Consistent flow pattern ≥ 84 (check each node). One number only per arc. No unnumbered arcs.

d1A1: CAO, showing flow of 102, must follow from their routes.

Mark scheme (e)

Scheme	Marks
The cut through AT_1 , DT_1 , DT_2 , DE, BE, CB and S_2C has value 102	DB1
Value of the flow is 102 so by max flow – min cut theorem flow is maximal	DB1
	(2)
	13 marks

Notes

e1DB1: Must have attempted (d) - at least one number on all but one arc, and made an attempt at a cut, either drawn or stated.

e2DB1: CSO - (d) fully correct (showing a correct flow of 102) and a correct cut. Must refer to max flow-min cut theorem – all four words.

2.

Mark scheme (a)

Scheme	Marks	AO
$C_1 (= 40 + 35 + 19 + 6) = 100$	B1	1.1b
$C_2 (= 13 + 12 + 16 - 2 - 3 + 43) = 79$	B1	1.1b
	(2)	

Notes

B1: CAO

B1: CAO

Mark scheme (b)

Scheme	Marks	AO
Deduces the maximum possible flow is ≤ 79 litres per second	B1ft	2.2a
	(1)	

Notes

B1ft: deduced from their least value given in (a) - must include 'less than or equal to' (oe)

Mark scheme (c)

Scheme	Marks	AO
Initial flow = 58	B1	1.1b
	(1)	

Notes

B1: CAO

Mark scheme (d)

Scheme	Marks	AO
e.g. SAET – 4, SADET – 2, SBGFT – 2, SCFT – 1	M1	1.1b
	A1	1.1b
	A1	1.1b
	(3)	

Notes

M1: One flow augmenting route found from S to T

A1: Two correct routes + flow values

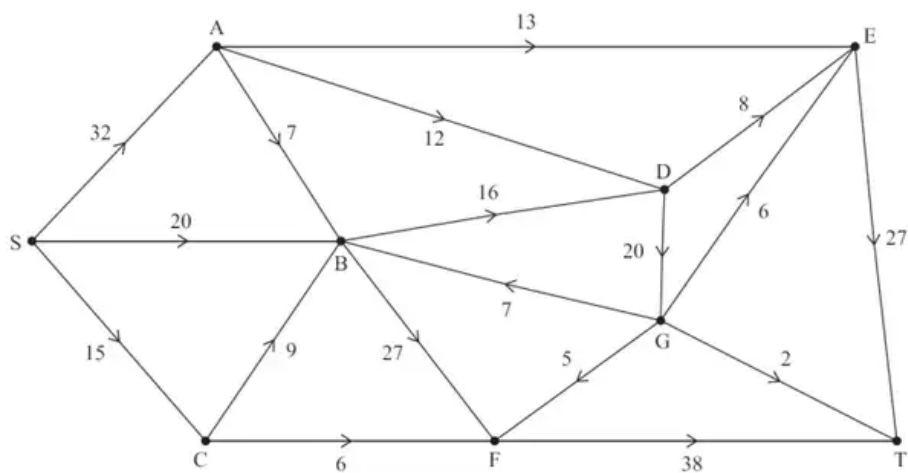
A1: CSO – increasing the flow by 9

Note possible flow augmenting routes are only $_ADET + 2_AET + 4_BGFT + 2_CFT + 1$ where $_$ represents either S, SB, SC or SCB

(SADGFT + 2 is also valid but this prevents DET being increased)

Mark scheme (e)

Scheme	Marks	AO
e.g.	B1	2.2a



(1)

Notes

B1: CAO – one number only per arc (only SA, SB, SC, AB and CB can vary depending on their flow augmenting routes)

Mark scheme (f)

Scheme	Marks	A0
Use of max-flow min-cut theorem	M1	2.1
Identification of cut through AE, AD, BD, DG, GE, GT, GF, BF and CF	A1	3.1a
Value of flow = 67	A1	2.2a
Therefore it follows that flow is maximal		
	(3)	
	(11 marks)	

Notes

M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through **saturated** arcs) (A valid cut through saturated arcs must either be listed or drawn)

May be drawn on either diagram or listed as arcs or set notation $\{S, A, B, C, G\} \{D, E, F, T\}$

A1: Use appropriate process of finding a minimum cut: cut + value correct and value of flow stated

A1: Correct deduction that the flow is maximal – must see all 4 words max flow min cut and conclusion - dependent on previous A mark