

QQQ – Decision 2 – Chapter 4 – Flows in Networks 2 (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1.

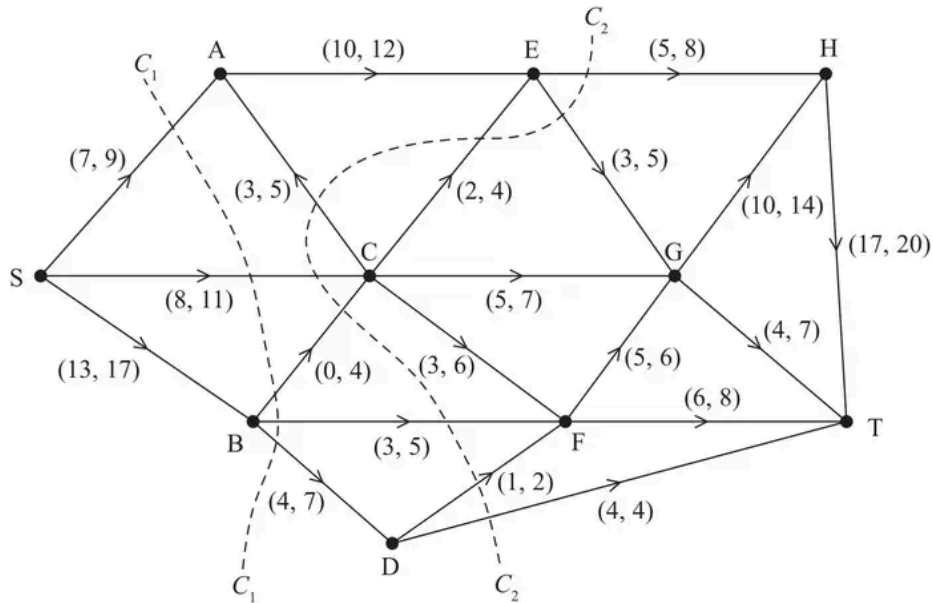


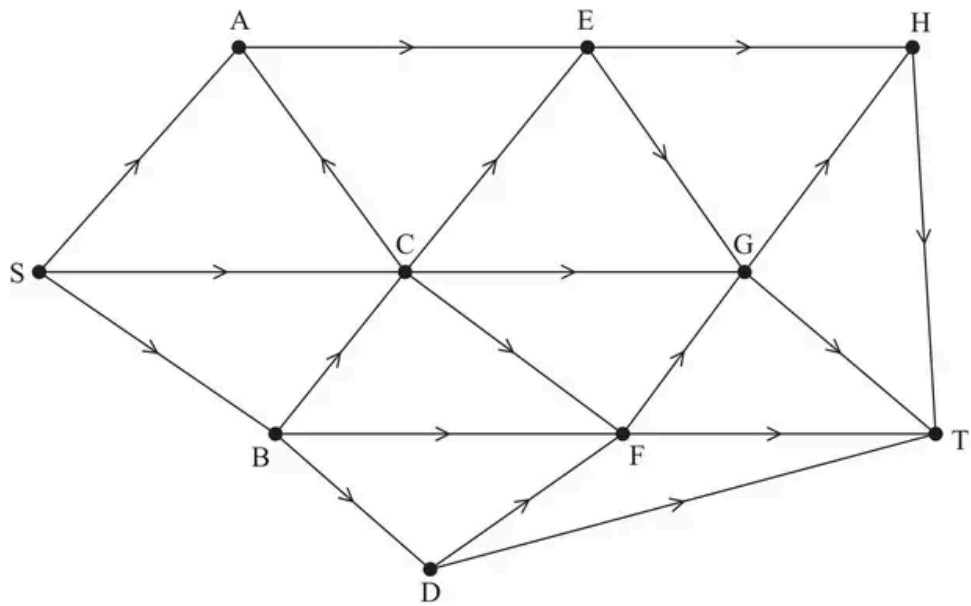
Figure 2

Figure 2 shows a capacitated, directed network. The network represents a system of pipes through which fluid flows from a source, S, to a sink, T.

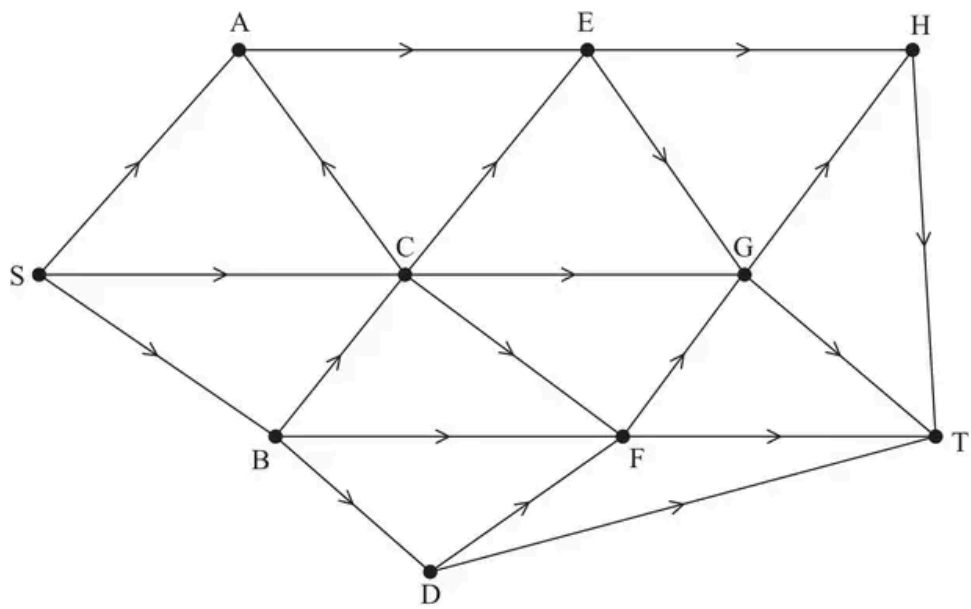
The numbers (l, u) on each arc represent, in litres per second, the lower capacity, l , and the upper capacity, u , of the corresponding pipe.

Two cuts C_1 and C_2 are shown.

- (a) Find the capacity of
 - (i) cut C_1
 - (ii) cut C_2 (2)
- (b) Explain why the arcs AE and CE cannot be at their upper capacities simultaneously. (1)
- (c) Explain why a flow of 31 litres per second through the system is not possible. (1)
- (d) Hence determine a minimum feasible flow and a maximum feasible flow through the system. You must draw these feasible flows on the diagrams below and give reasons to justify your answer. **You should not apply the labelling procedure to find these flows.** (4)



Minimum feasible flow



Maximum feasible flow

(Total 8 marks)

2.

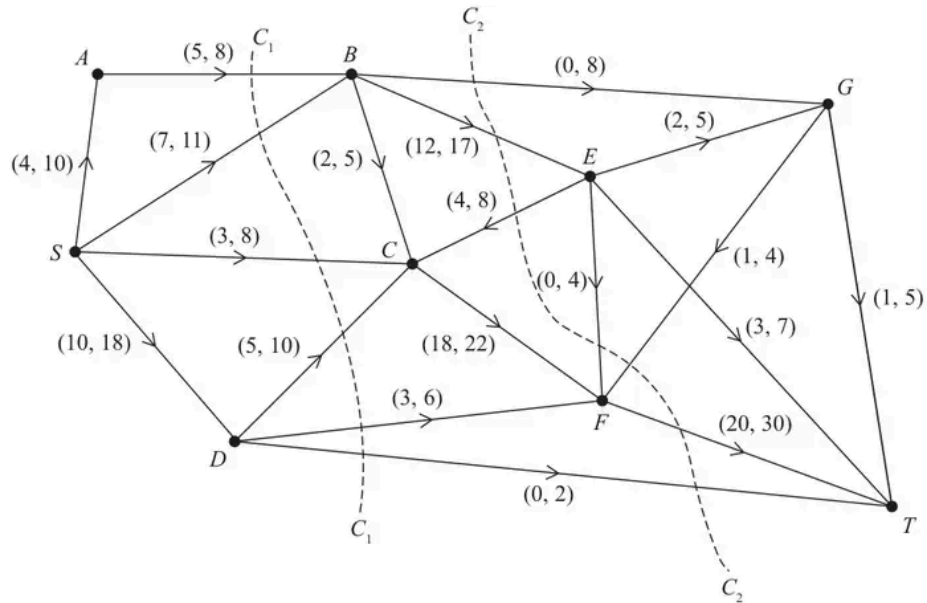


Figure 1

Figure 1 shows a capacitated, directed network. The network represents a system of pipes through which fluid can flow.

The weights on the arcs show the lower and upper capacities for the corresponding pipes, in litres per second.

(a) Calculate the capacity of

(i) cut C_1

(ii) cut C_2 (2)

(b) Using only the capacities of cuts C_1 and C_2 , state what can be deduced about the maximum flow through the system. (1)

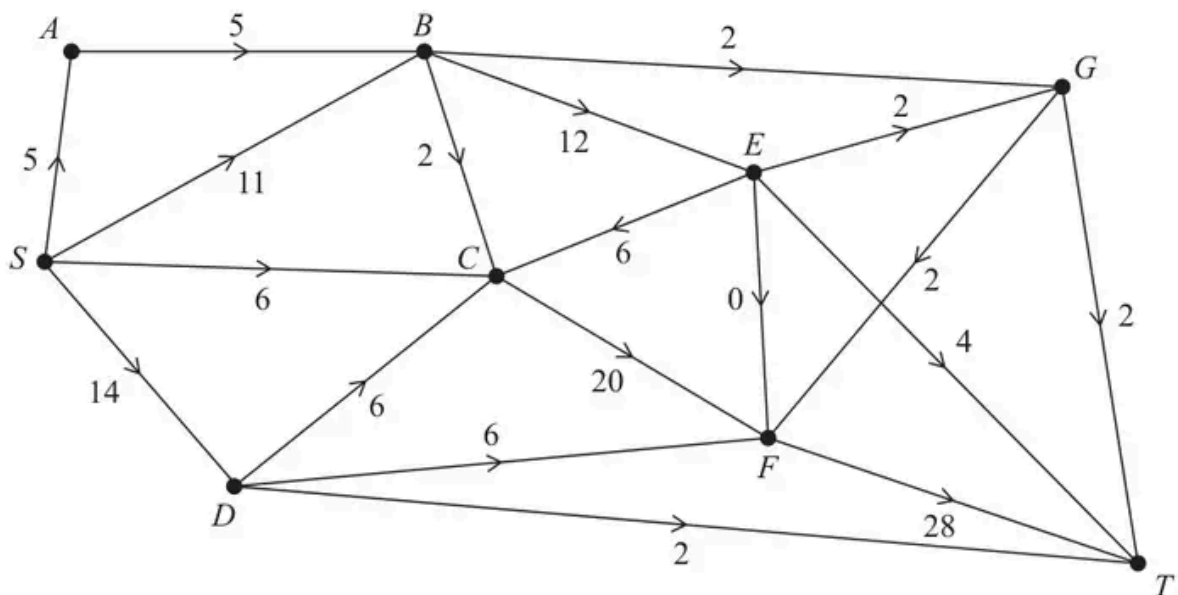


Figure 2

Figure 2 shows an initial flow through the same network.

- (c) State the value of the initial flow. (1)
- (d) By entering values along BC , CF and DT , complete the labelling procedure on Diagram 1 below. (2)

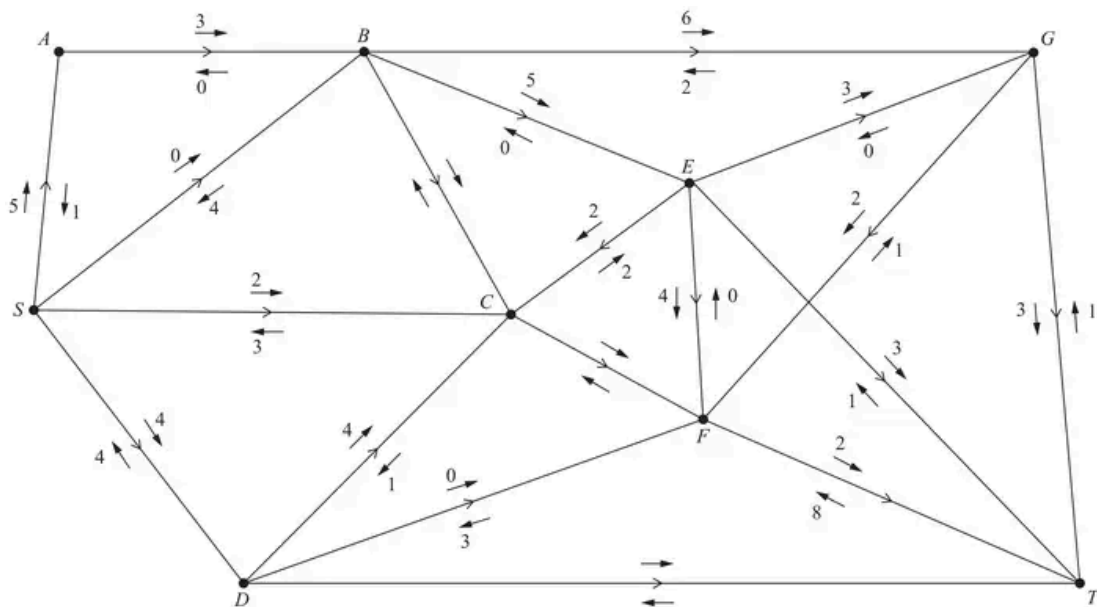


Diagram 1

- (e) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow. (3)
- (f) Use your answer to (e) to find a maximum flow pattern for this system of pipes and draw it on Diagram 2 below. (1)

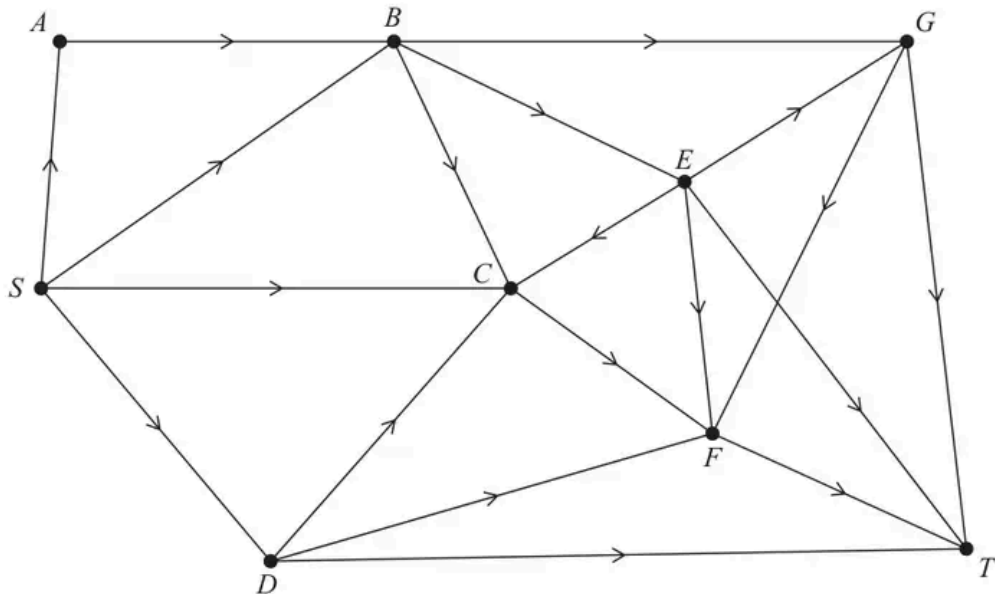


Diagram 2

- (g) Prove that the answer to (f) is optimal. (3)

A vertex restriction is now applied to B so that no more than 16 litres per second can flow through it.

- (h) (i) Complete Diagram 3 below to show this restriction. (3)
(ii) State the value of the maximum flow with this restriction. (3)

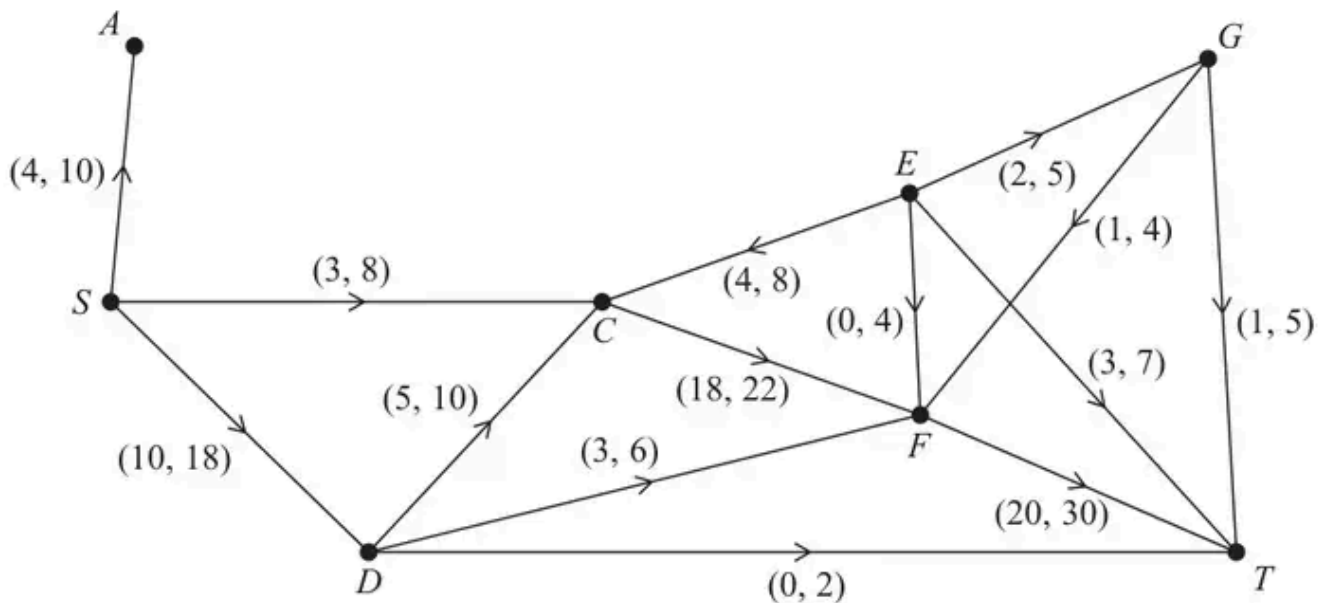


Diagram 3

(Total 16 marks)