

QQQ – Decision 2 – Chapter 4 – Flows in Networks 2 (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1.

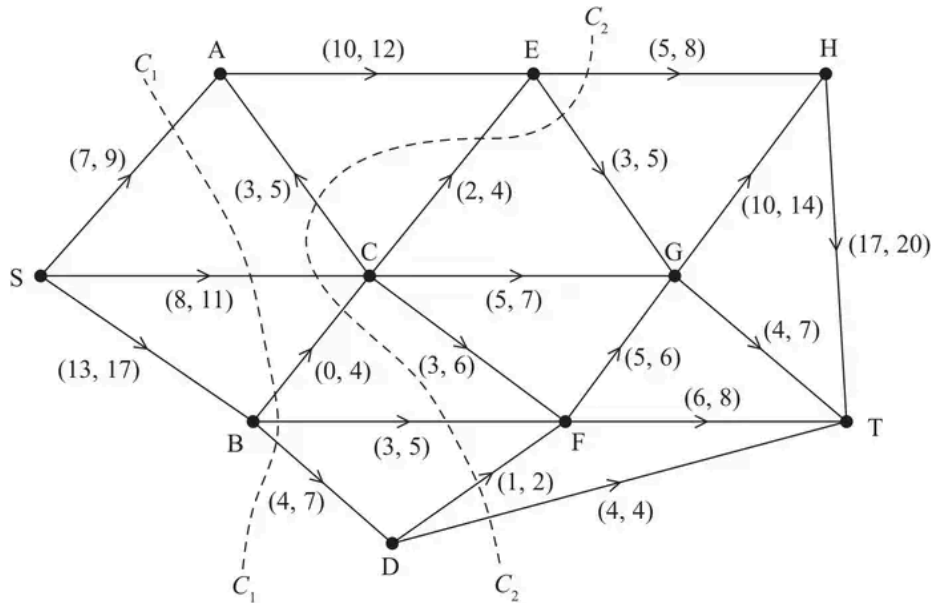


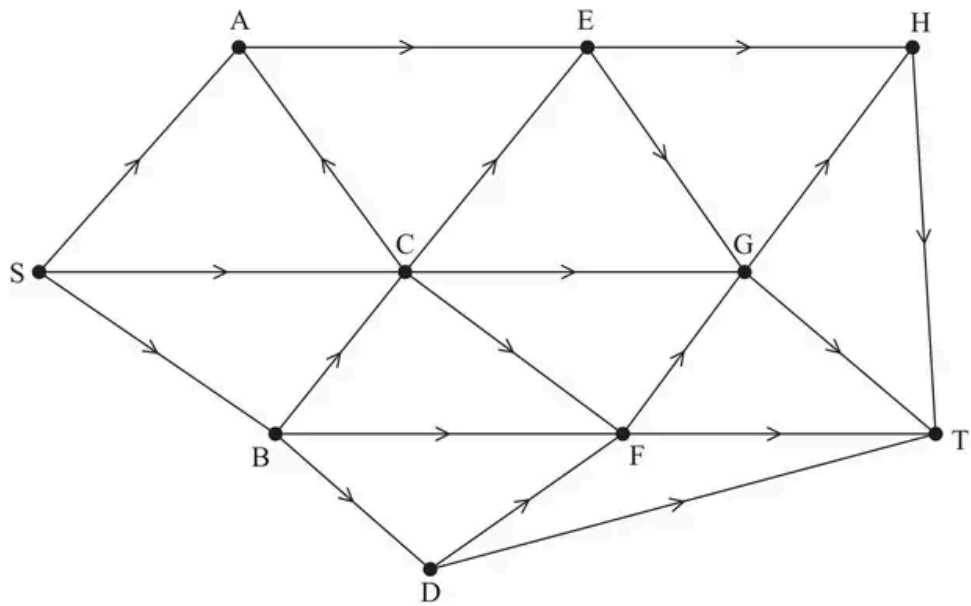
Figure 2

Figure 2 shows a capacitated, directed network. The network represents a system of pipes through which fluid flows from a source, S, to a sink, T.

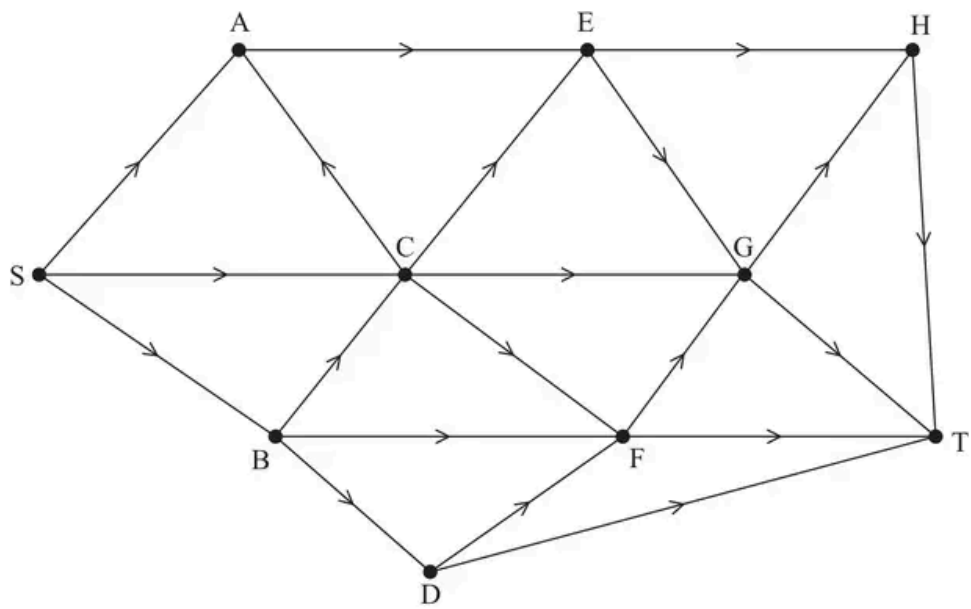
The numbers (l, u) on each arc represent, in litres per second, the lower capacity, l , and the upper capacity, u , of the corresponding pipe.

Two cuts C_1 and C_2 are shown.

- (a) Find the capacity of
 - (i) cut C_1
 - (ii) cut C_2 (2)
- (b) Explain why the arcs AE and CE cannot be at their upper capacities simultaneously. (1)
- (c) Explain why a flow of 31 litres per second through the system is not possible. (1)
- (d) Hence determine a minimum feasible flow and a maximum feasible flow through the system. You must draw these feasible flows on the diagrams below and give reasons to justify your answer. **You should not apply the labelling procedure to find these flows.** (4)



Minimum feasible flow



Maximum feasible flow

(Total 8 marks)

2.

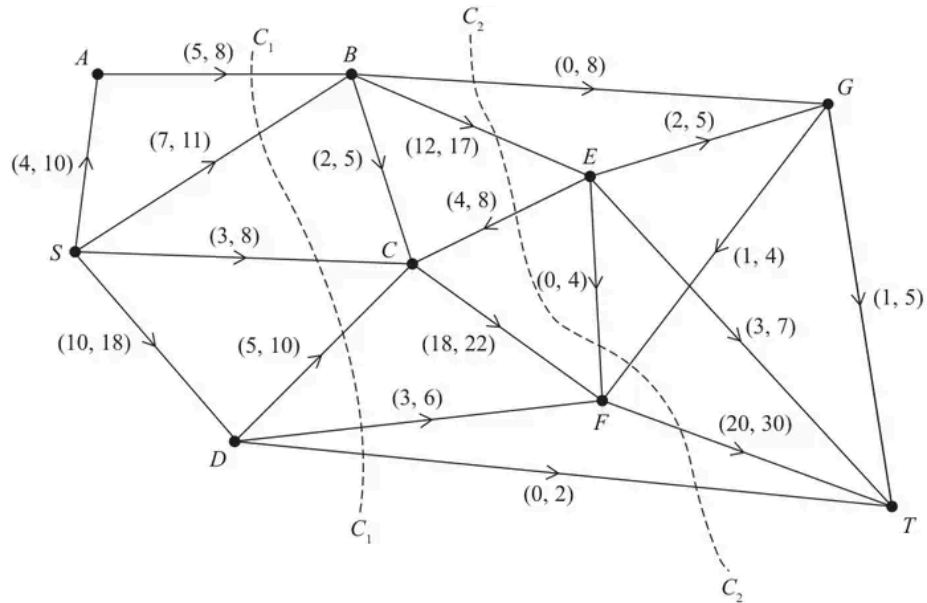


Figure 1

Figure 1 shows a capacitated, directed network. The network represents a system of pipes through which fluid can flow.

The weights on the arcs show the lower and upper capacities for the corresponding pipes, in litres per second.

(a) Calculate the capacity of

(i) cut C_1

(ii) cut C_2 (2)

(b) Using only the capacities of cuts C_1 and C_2 , state what can be deduced about the maximum flow through the system. (1)

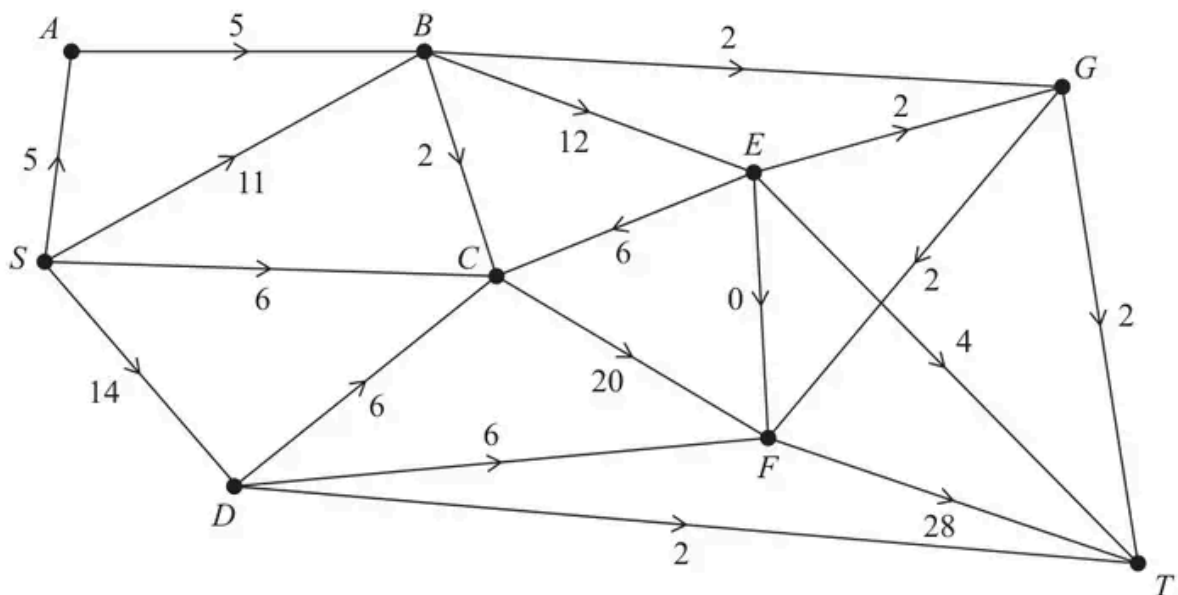


Figure 2

Figure 2 shows an initial flow through the same network.

(c) State the value of the initial flow. (1)

(d) By entering values along BC , CF and DT , complete the labelling procedure on Diagram 1 below. (2)

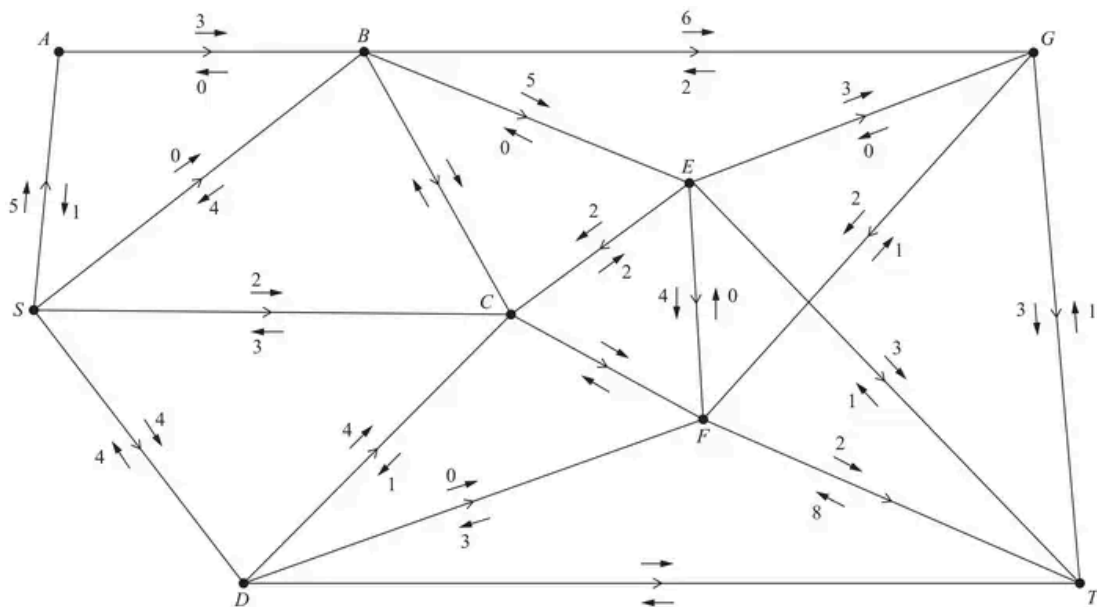


Diagram 1

(e) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow. (3)

(f) Use your answer to (e) to find a maximum flow pattern for this system of pipes and draw it on Diagram 2 below. (1)

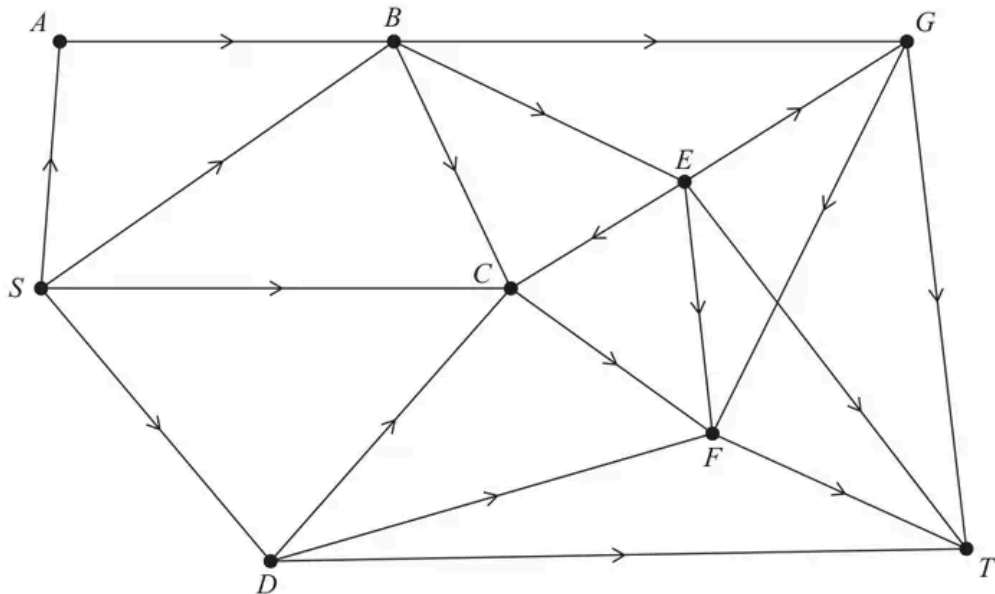


Diagram 2

- (g) Prove that the answer to (f) is optimal. (3)

A vertex restriction is now applied to B so that no more than 16 litres per second can flow through it.

- (h) (i) Complete Diagram 3 below to show this restriction. (3)
(ii) State the value of the maximum flow with this restriction. (3)

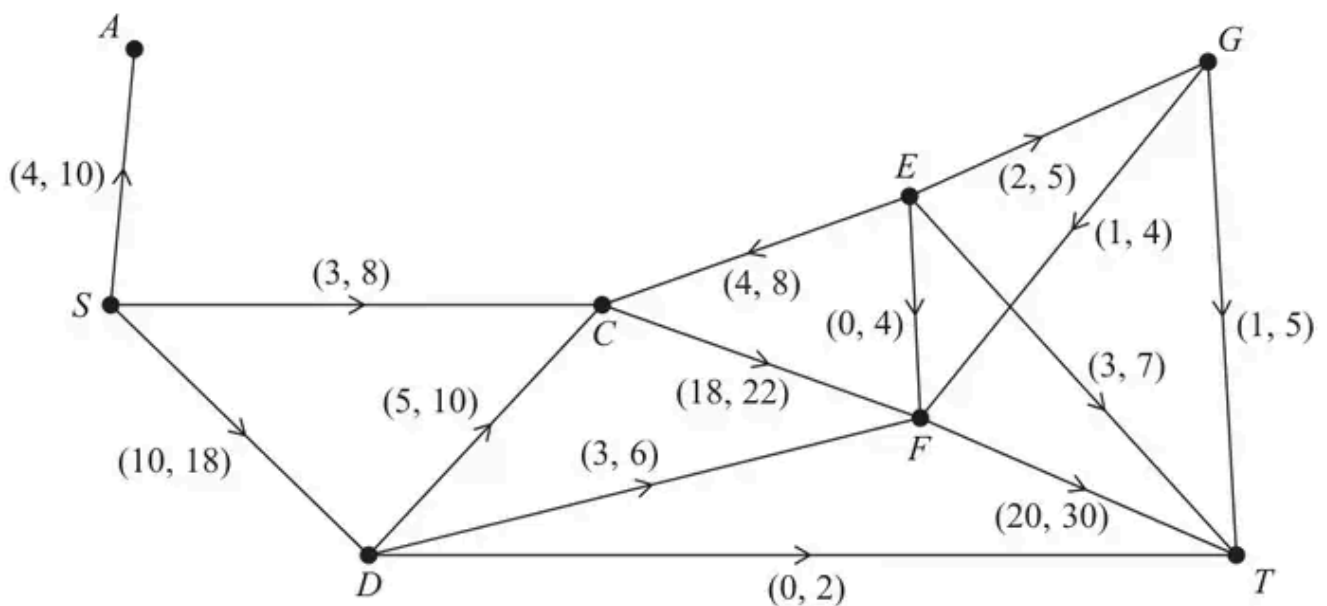


Diagram 3

(Total 16 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
(i) $C_1 = 9 + 11 + 4 + 5 + 7 = 36$	1B1	1.1b
(ii) $C_2 = 8 + 5 - 2 - 3 + 11 + 4 + 5 + 2 + 4 = 34$	2B1	1.1b
	(2)	

Notes

(a)(i) 1B1: CAO

(ii) 1B1: CAO

Mark scheme (b)

Scheme	Marks	AO
If AE and CE were both full to capacity then $12 + 4 = 16$ litres per second would flow through E but the maximum capacity of the two arcs out of E (EH and EG) is only $8 + 5 = 13$ so AE and CE cannot both be full to capacity	1B1	2.4
	(1)	

Notes

1B1: Calculates maximum capacity entering E and compares with maximum capacity leaving E. Concludes that maximum capacity into E exceeds maximum capacity out of E.

Condone statements such as 'flow into E = $12 + 4 < 13$ which is the flow out of E'.

Mark scheme (c)

Scheme	Marks	AO
The minimum flow through the arcs AE, CE, CG, FG, FT and DT (which provides a cut for the network) is $10 + 2 + 5 + 5 + 6 + 4 = 32$ so a minimum of 32 must be flowing through the system so 31 is not possible.	1B1	2.4

Notes

1B1: valid reason why the flow in the network cannot be 31 litres per second.

Note: If smallest of C_1 and C_2 in a) is less than 31 then DO NOT allow this mark for deducing that '31 > {smallest of answers from a} hence, flow of 31 is not possible'.

Mark scheme (d)

Scheme	Marks	AO
Attempt to find a flow of 32 using the answer to (c)	1M1	3.4
E.g. Minimum flow of 32 	1A1	1.1b
Attempt to augment minimum flow and recognise that from (a) the maximum flow is less than or equal to 34	1B1	2.1
E.g. Maximum flow of 34	2B1	1.1b

	(4)	
	(8 marks)	

Notes

1M1: Award this mark for either:

- Indicates 'minimum flow > 31 so minimum flow could be 32', OR
- Attempts to find flow of 32 in which: the sum of flows along arcs from S is 32 **and** $7 \leq \text{flow along SA} \leq 9, 8 \leq \text{flow along SC} \leq 11$ **and** $13 \leq \text{flow along SB} \leq 17$, OR
- Attempts to find flow of 32 in which: the sum of flows along arcs into T is 32 **and** flow along DT = 4, $6 \leq \text{flow along FT} \leq 8, 4 \leq \text{flow along GT} \leq 7$ **and** $17 \leq \text{flow along HT} \leq 20$ (corrected from the printed mark scheme, which gives the upper bound for GT as 4)
- Identifies both 'min flow = 32' **AND** 'max flow = 34'

Note: Only need consider arcs incident to S for the second bullet point above, or arcs incident to T for the third bullet point. Flows along other arcs may be incorrect or missing.

1A1: CAO for consistent flow of 32. One number per arc. Check for consistency at each node.

1B1: States that maximum flow must be 34 and makes some reference to (smallest cut in) part a).

1B1: CAO for consistent flow of 34. One number per arc. Check for consistency at each node.

2.

Mark scheme (a)

Scheme	Marks	AO
--------	-------	----

(i) $C_1 = 8 + 11 + 8 + 10 + 6 + 2 = 45$	B1	1.1b
(ii) $C_2 = 8 + 17 - 4 - 0 - 1 + 30 + 2 = 52$	B1	1.1b
	(2)	

Notes

(i) B1: cao

(ii) B1: cao

Mark scheme (b)

Scheme	Marks	AO
Deduces the maximum possible flow is ≤ 45 litres per second	B1ft	2.2a
	(1)	

Notes

B1ft: Deduced from their least value given in (a) - must include 'less than or equal to' (oe)

Mark scheme (c)

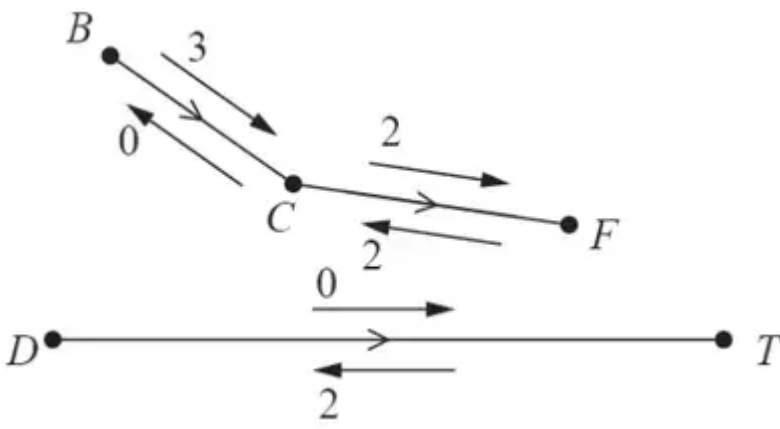
Scheme	Marks	AO
Initial flow = 36	B1	1.1b
	(1)	

Notes

B1: cao

Mark scheme (d)

Scheme	Marks	AO
--------	-------	----

	M1	1.1b
	A1	1.1b
	(2)	

Notes

M1: Two numbers on each arc and at least two arcs or four numbers correct (so correct numbers with the correct arrows)

A1: can do give bod since they might well cross these numbers out

Mark scheme (e)

Scheme	Marks	AO
e.g., SABGT – 2, SCFT – 2, SDCET – 2, SABET – 1	M1	1.1b
e.g., SABGT – 3, SCFT – 2, SDCET – 2	A1	1.1b
	A1	1.1b
	(3)	

Notes

M1: One flow augmenting route found from S to T

A1: Two correct routes + flow values

A1: cso – increasing the flow by 7

Mark scheme (f)

Scheme	Marks	AO
e.g.	B1	2.2a

	(1)	

Notes

Bl: cao

Mark scheme (g)

Scheme	Marks	AO
Use of max-flow min-cut theorem	M1	2.1
Identification of cut through AB, SB, BC, EC, CF, DF and DT	A1	3.1a
Value of flow = 43	A1	2.2a
Therefore it follows that flow is maximal		
	(3)	

Notes

M1: Construct argument based on max-flow min-cut theorem (e.g., attempt to find a cut through saturated arcs)

A1: Use appropriate process of finding a minimum cut: cut + value correct

A1: Correct deduction that the flow is maximal

Mark scheme (h)

Scheme	Marks	AO
--------	-------	----

