

QQQ – Decision 2 – Chapter 3 – Flows in Networks 1 (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1.

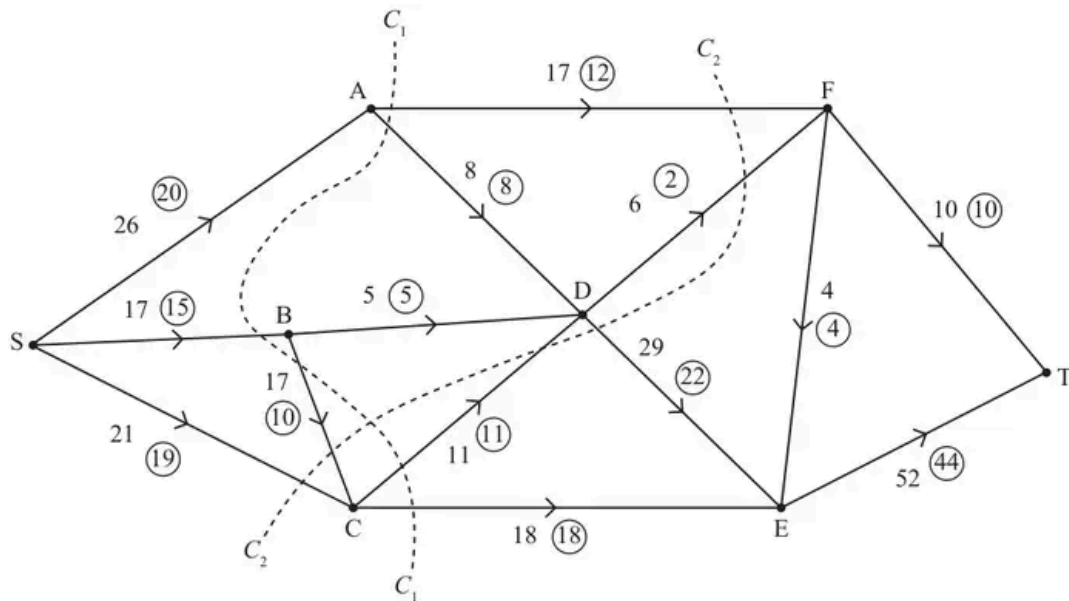


Figure 1

Figure 1 shows a capacitated, directed network of pipes. The number on each arc represents the capacity of that pipe. The numbers in circles represent a feasible flow from S to T.

- State the value of the feasible flow. (1)
- Find the capacity of cut C_1 and the capacity of cut C_2 . (2)
- By inspection, find a flow-augmenting route to increase the flow by two units. You must state your route. (1)
- Prove that, once the flow-augmenting route found in (c) has been applied, the flow is now maximal. (3)

(Total 7 marks)

2.

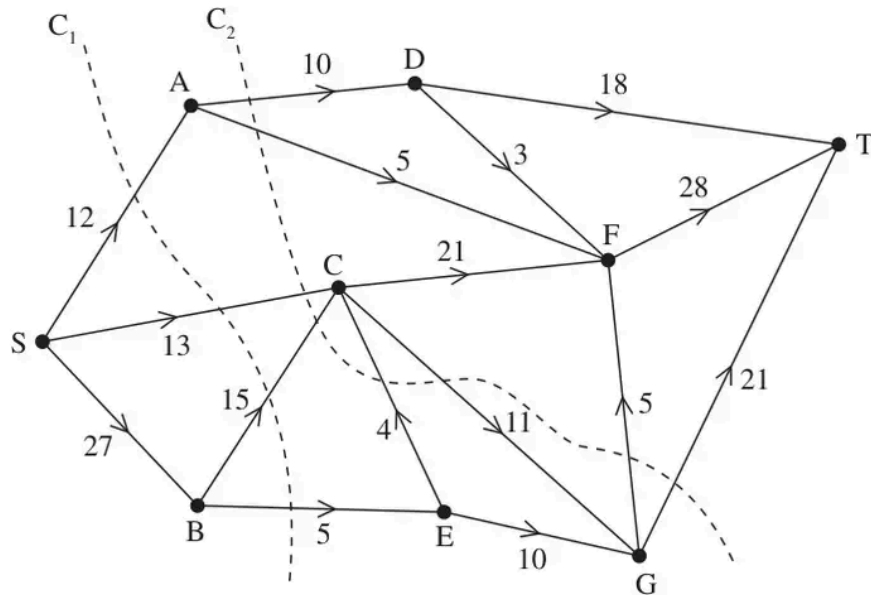


Figure 1

Figure 1 shows a capacitated network. The capacity of each arc is shown on the arc. Two cuts C_1 and C_2 are shown.

- (a) Find the capacity of each of the two cuts. (2)

Given that one of these two cuts is a minimum cut,

- (b) write down the maximum possible flow through the network. (1)

Given that the network now has a maximal flow from S to T,

- (c) determine the flow along arc SB. (1)

- (d) Explain why arcs GF and GT cannot both be saturated. (1)

Given that arcs EC, AD and DF are saturated and that there is no flow along arc GF,

- (e) determine a maximum flow pattern for this network and draw it on Diagram 1 in the answer book. You do not need to use the labelling procedure to determine the maximum flow. (2)

(Total 7 marks)

3.

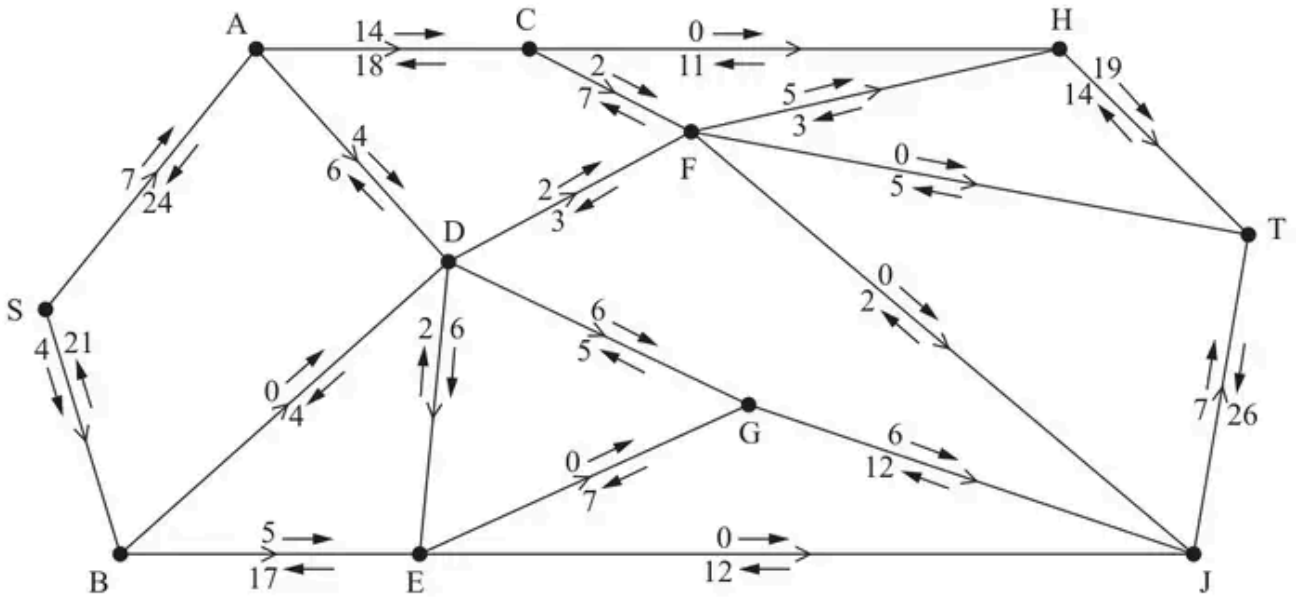


Figure 1

Alexa is monitoring a system of pipes through which fluid can flow from the source, S, to the sink, T. Currently, fluid is flowing through the system from S to T.

Alexa initialises the labelling procedure for this system, and the excess capacities and potential backflows are shown on the arrows either side of each arc, as shown in Figure 1.

- (a) State the value of the initial flow. (1)
- (b) Explain why arcs DF and DG can never both be full to capacity. (1)
- (c) Obtain the capacity of the cut that passes through the arcs AC, AD, BD, DE, EG and EJ. (1)
- (d) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow. (3)
- (e) Use your answers to part (d) to find a maximum flow pattern for this system of pipes and draw it on Diagram 1 in the answer book. (1)

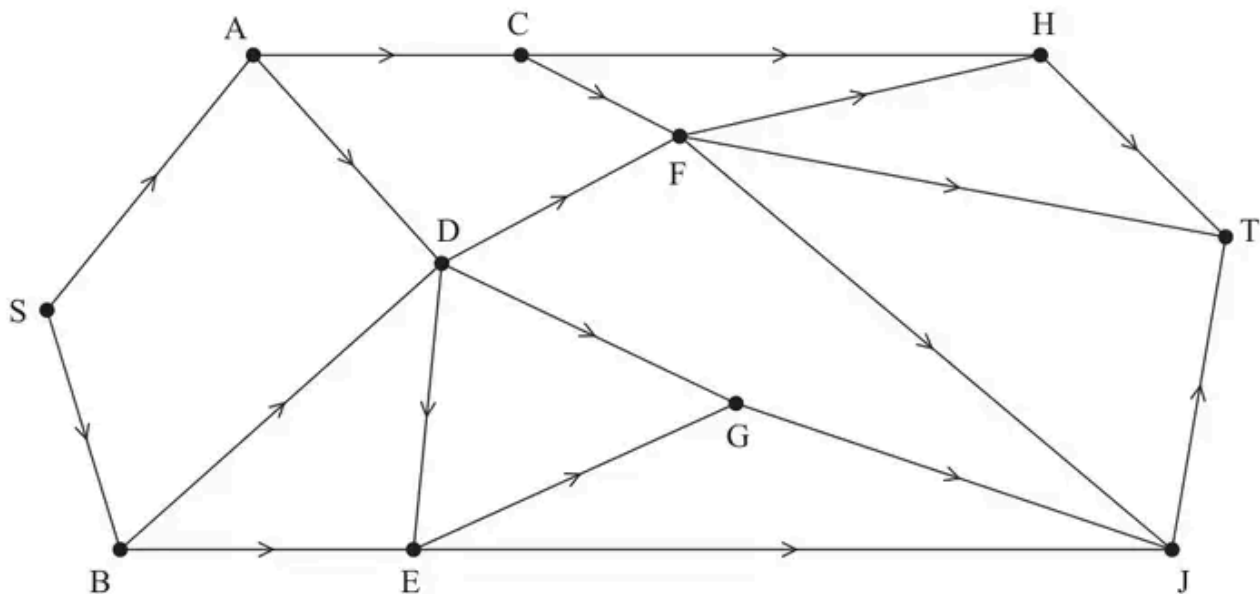


Diagram 1

(f) Prove that the answer to part (e) is optimal.

(3)

(Total 10 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
54	B1	1.1b
	(1)	

Notes

B1: CAO

Mark scheme (b)

Scheme	Marks	AO
$C_1 (= 17 + 8 + 17 + 11 + 18) = 71$	B1	1.1b
$C_2 (= 17 + 6 + 29 + 17 + 21) = 90$	B1	1.1b
	(2)	

Notes

B1: CAO for C_1

B1: CAO for C_2

Mark scheme (c)

Scheme	Marks	AO
SAFDET	B1	1.1b
	(1)	

Notes

B1: CAO

Mark scheme (d)

Scheme	Marks	AO
--------	-------	----

Use of max-flow min-cut theorem		
Identification of cut through FT, FE, DF, AD, BD, CD and CE	M1	2.1
Value of flow = 56	A1	3.1a
Therefore it follows that flow is maximal	A1	2.2a
	(3)	
	(7 marks)	

Notes

M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through arcs – must contain source on one side and sink on the other). Allow a cut drawn on the diagram (need not be the correct one). If cut not drawn, must list arcs not values.

A1: Use appropriate process of finding a minimum cut – FT, FE, DF, AD, BD, CD and CE plus value correct and value of flow through the network stated correctly (56)

A1: Correct deduction that the flow is maximal – must use all four words ‘maximum’, ‘flow’, ‘minimum’ and ‘cut’ (allow use of max and min) dependent on previous A1.

2.

Mark scheme (a)

Scheme	Marks
$C_1 = 45, C_2 = 73$	B1 B1
	(2)

Notes

a1B1: CAO for C_1 (45)

a2B1: CAO for C_2 (73)

Mark scheme (b)

Scheme	Marks
45	B1ft
	(1)

Notes

b1B1ft: 45 or the value of their smallest cut from (a)

Mark scheme (c)

Scheme	Marks
20	B1
	(1)

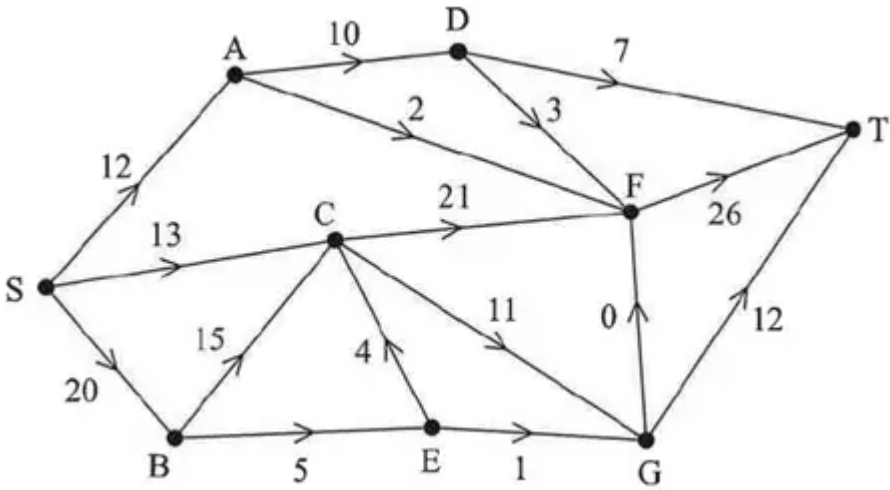
Notes
c1B1: CAO (20)

Mark scheme (d)

Scheme	Marks
The maximum capacity of the arcs flowing into G is 21 and so both GF and GT cannot be full to capacity as the capacity of the arcs flowing out of G is 26	B1
	(1)

Notes
d1B1: CAO – argument must be numerical in nature (as a minimum accept $26 > 21$ (oe))

Mark scheme (e)

Scheme	Marks
	M1 A1
	(2)
	7 marks

Notes

e1M1: Consistent flow pattern – check each node, must have exactly 1 number per arc (arc EC must be 4, AD – 10 and DF – 3 but all other arcs may have over-capacitated values)

e1A1: CAO

3.

Mark scheme (a)

Scheme	Marks	AO
45	B1	1.1b
	(1)	

Notes

B1: CAO (45)

Mark scheme (b)

Scheme	Marks	AO
e.g. the total capacity of arcs DF and DG is $5 + 11 = 16$. The capacity of the two arcs leading into D are 10 (from AD) and 4 (from BD) giving a total capacity into D of 14. As $14 < 16$ arcs DF and DG cannot both be full to capacity	B1	2.4
	(1)	

Notes

B1: CAO (as a minimum accept mention that the max flow into D is 14 and max flow out of D is 16 together with comparison of these two values – (node) D must be mentioned)

Mark scheme (c)

Scheme	Marks	AO
Value of cut = $32 + 10 + 4 + 7 + 12 = 65$	B1	1.1b
	(1)	

Notes

B1: CAO (65)

Mark scheme (d)

Scheme	Marks	AO
e.g. SACFHT – 2; SADGJT – 4; SBEDFHT – 2	M1	1.1b
e.g. SACFHT – 2; SADGJT – 2; SADFHT – 2; SBEDGJT – 2	A1	1.1b
e.g. SACFHT – 2; SADGJT – 4; SBEDGJT – 2	A1	1.1b
	(3)	

Notes

M1: One correct flow augmenting route found from S to T + flow value **or** two correct routes

A1: Two correct routes + correct flow values

A1: CSO – increasing the flow by 8 only

Mark scheme (e)

Scheme	Marks	AO
e.g. Alternative: DG 11, GJ 18, JT 32 DF 3, FH 5, HT 16	B1	1.1b
	(1)	

Notes

B1: CAO – condone more than one value on an arc only if one of these values is circled – mark those that have been circled only

Mark scheme (f)

Scheme	Marks	AO
--------	-------	----

Use of max-flow min-cut theorem	M1	2.1
Identification of cut through CH, CF, AD, BD, DE, EG and EJ		
Value of flow = 53	A1	3.1a
Therefore by the max-flow min-cut theorem it follows that flow is maximal	A1	2.2a
	(3)	
	(10 marks)	

Notes

M1: Construct the start of an argument based on the max-flow min-cut theorem (that is an attempt to find a genuine cut together with the value of either their cut or flow (but not re-iterating the cut given in (c) – AC, AD, BD, DE, EG, EJ))

A1: Use appropriate process of finding a minimum cut – must see correct cut + value correct (accept ‘53’ and the cut either stated or drawn on either diagram)

A1: Correct deduction that the flow is maximal by stating ‘maximum flow (equal to) minimum cut’ – dependent on previous A mark and the correct flow in (e)