

QQQ – Decision 2 – Chapter 3 – Flows in Networks 1 (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1.

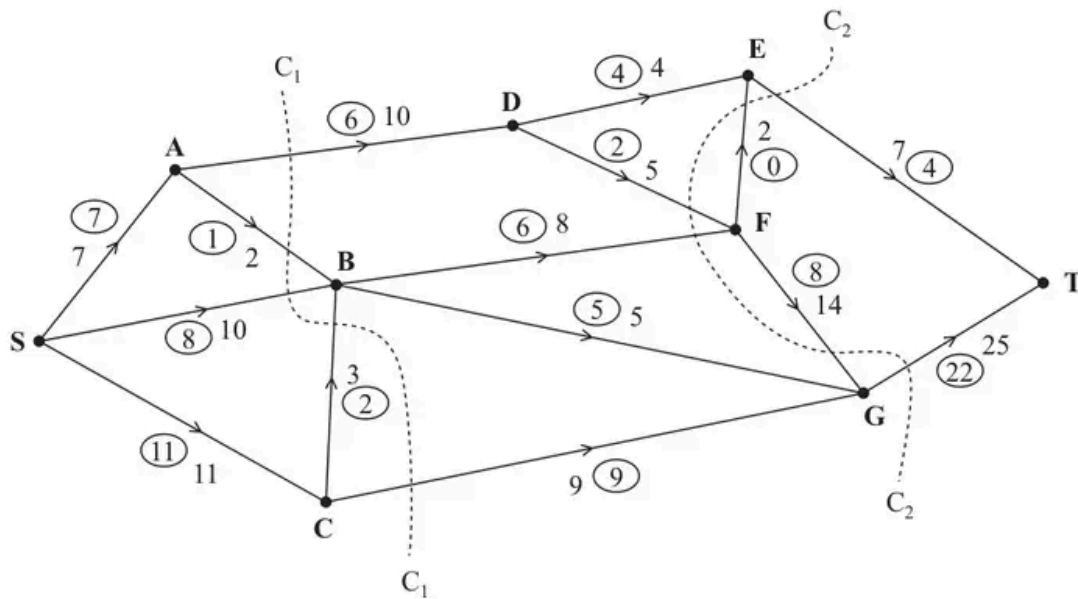


Figure 1

Figure 1 shows a capacitated network. The capacity of each arc is shown on the arc. The numbers in circles represent an initial flow from S to T.

Two cuts C_1 and C_2 are shown in Figure 1.

- (a) Find the capacity of each of the two cuts. (2)
- (b) Find the maximum flow through the network. You must list each flow-augmenting route you use together with its flow. (3)

(Total 5 marks)

2.

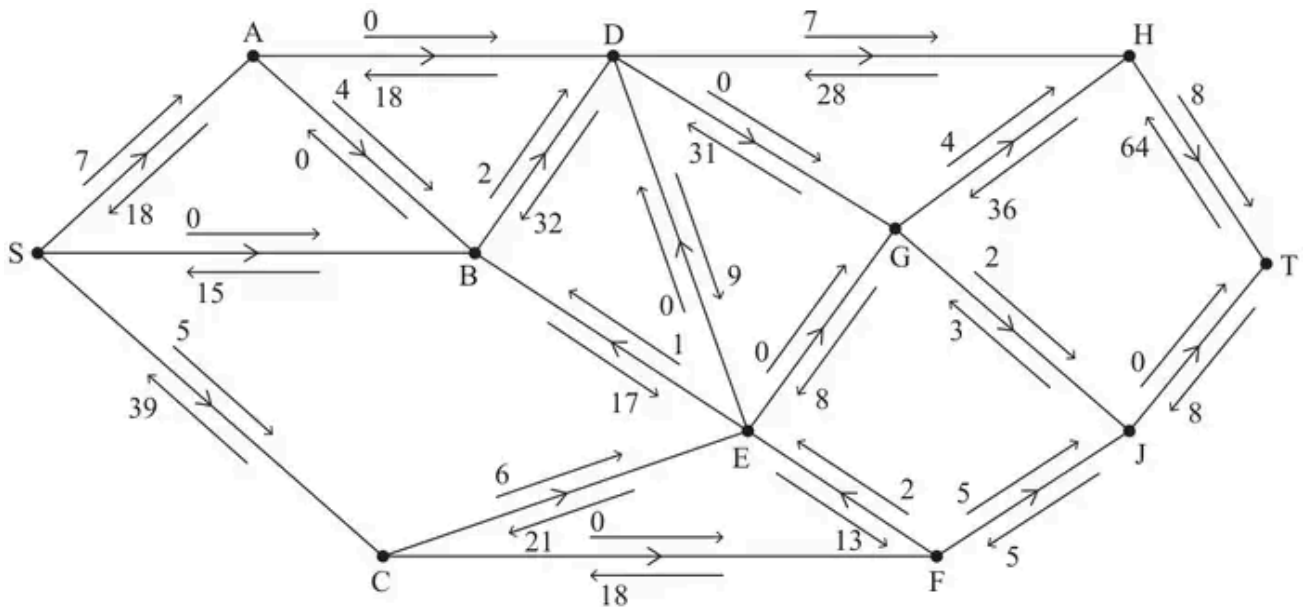


Figure 1

An engineer monitors a system of pipes through which a fluid flows from the source, S, to the sink, T.

The engineer initialises the labelling procedure for this system, and the excess capacities and potential backflows are shown on the arrows either side of each arc, as shown in Figure 1.

- (a) State the value of the initial flow. (1)
- (b) Obtain the capacity of the cut that passes through the arcs SA, SB, CE, FE and FJ. (1)
- (c) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow. (3)
- (d) Use your answer to (c) to draw a maximum flow pattern on Diagram 1 in the answer book. (1)

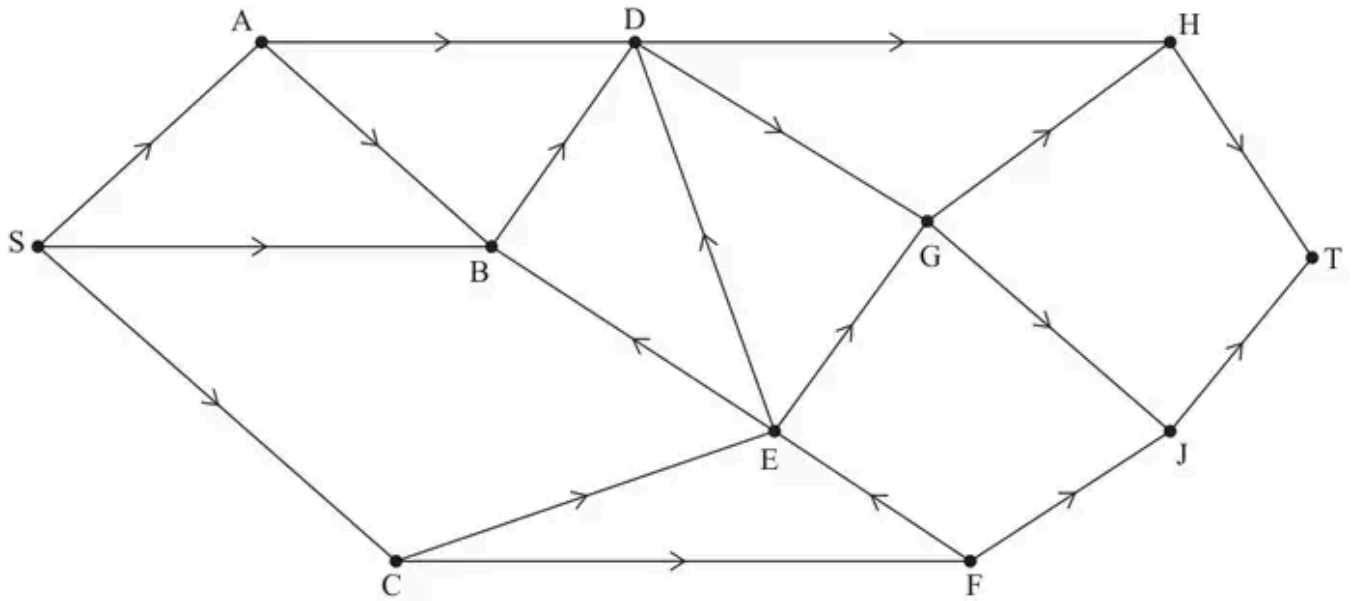


Diagram 1

(e) Prove that the answer to (d) is optimal.

(3)

(Total 9 marks)

3.

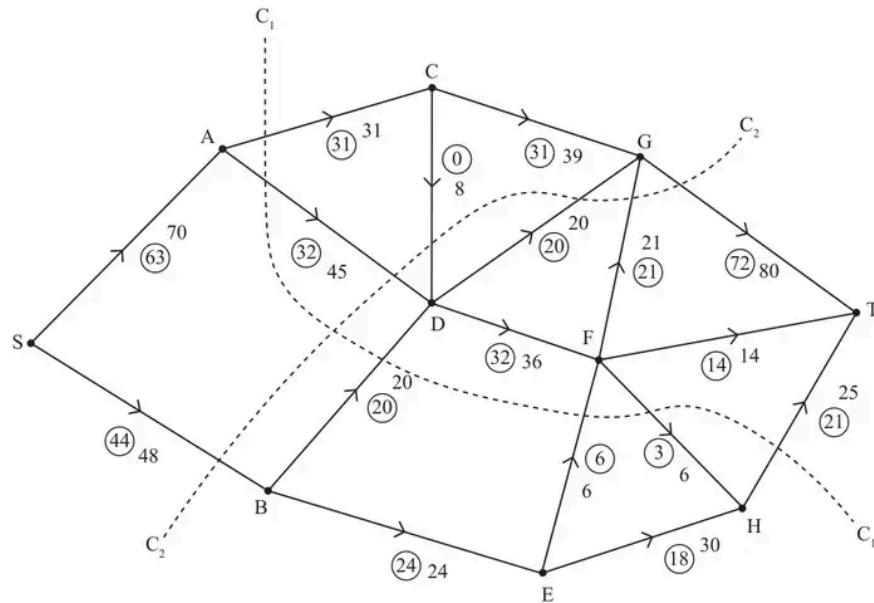


Figure 1

Figure 1 shows a capacitated, directed network of pipes. The number on each arc represents the capacity of the corresponding pipe. The numbers in circles represent a feasible flow from S to T.

- (a) State the two conditions satisfied by a feasible flow. (2)
- (b) List the seven saturated arcs in Figure 1. (1)
- (c) Find the capacity of
 - (i) cut C_1
 - (ii) cut C_2 (2)
- (d) Write down a flow-augmenting route that increases the flow by four units. (1)
- (e) Use the answer to part (d) to draw the resulting flow pattern on Diagram 1 in the answer book. (2)

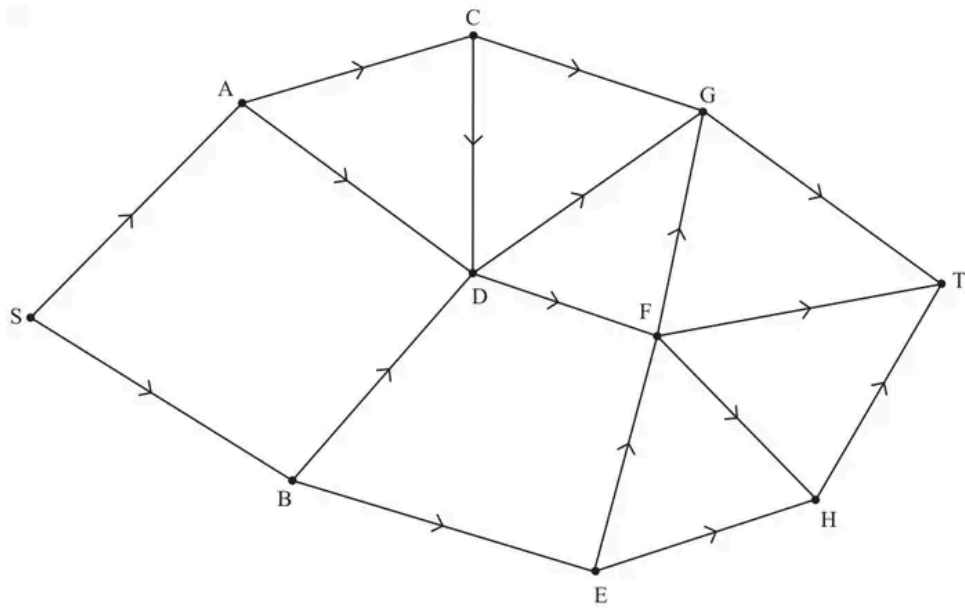


Diagram 1

(f) Prove that the answer to part (e) is a maximum flow.

(3)

(Total 11 marks)