

QQQ – Decision 2 – Chapter 1 – Transportation Problems (Version 2)

Total Marks: 23

(23 = Platinum, 21 = Gold, 18 = Silver, 16 = Bronze)

1. Three warehouses, P, Q and R, supply washing machines to four retailers, A, B, C and D. The table gives the cost, in pounds, of transporting a washing machine from each warehouse to each retailer. It also shows the number of washing machines held at each warehouse and the number of washing machines required by each retailer. The total cost of transportation is to be minimised.

	A	B	C	D	Supply
P	11	22	13	17	25
Q	21	8	19	14	27
R	15	10	9	12	28
Demand	18	16	20	26	

Formulate this transportation problem as a linear programming problem. You must define your decision variables and make the objective function and constraints clear.

You do not need to solve this problem.

(7)

(Total 7 marks)

2. Table 1 shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to each of four demand points, 1, 2, 3 and 4. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution to this transportation problem is required.

	1	2	3	4	Supply
A	24	32	21	34	27
B	28	31	29	37	41
C	25	41	33	35	31
D	23	32	31	36	14
Demand	33	35	25	20	

Table 1

Table 2 shows an initial solution given by the north-west corner method.

	1	2	3	4
A	27			
B	6	35		
C		0	25	6
D				14

Table 2

- (a) Explain why a zero has been placed in cell C2 in Table 2. State the other cell in Table 2 in which the zero could have been placed. (2)
- (b) State the shadow costs clearly and enter the improvement indices into Table 3 in your answer book. (3)

Taking the most negative improvement index to indicate the entering cell,

- (c) list the stepping-stone route that should be used to obtain the next solution. You should make clear the cells that are included in your route and state your entering and exiting cells. [You do **not** need to state the next solution. You do **not** need to solve this problem.] (2)

(Total 7 marks)

3. The table below shows the cost of transporting one unit of stock from each of four supply points, 1, 2, 3 and 4, to each of three demand points, A, B and C. It also shows the stock held at each supply point and the stock required at each demand point. A minimal cost solution is required.

	A	B	C	Supply
1	31	29	32	20
2	22	33	27	22
3	25	27	32	20
4	23	26	38	38
Demand	35	25	30	

(a) Add a dummy demand point and appropriate values to Table 1 in the answer book. (1)

Table 2 shows an initial solution given by the north-west corner method.

Table 3 shows some of the improvement indices for this solution.

	A	B	C	D
1	20			
2	15	7		
3		18	2	
4			28	10

Table 2

	A	B	C	D
1		-13		-9
2			-11	
3				
4	1	-7		

Table 3

(b) Calculate the shadow costs and the missing improvement indices and enter them into Table 3 in the **answer book**. (4)

- (c) Taking the most negative improvement index to indicate the entering square, use the stepping-stone method once to obtain an improved solution. You must make your route clear and state your entering cell and exiting cell. (4)

(Total 9 marks)

Solutions

1.

Scheme	Marks
Let x_{ij} be the number of washing machines transported from i to j where $i \in \{P, Q, R\}$ and $j \in \{A, B, C, D\}$	B1
The objective is to minimise $C = 11x_{PA} + 22x_{PB} + 13x_{PC} + 17x_{PD} + 21x_{QA} + 8x_{QB} + 19x_{QC} + 14x_{QD} + 15x_{RA} + 10x_{RB} + 9x_{RC} + 12x_{RD}$	B1 B1
Subject to $x_{PA} + x_{PB} + x_{PC} + x_{PD} = 25 \text{ or } \sum x_{Pj} = 25$	M1
$x_{QA} + x_{QB} + x_{QC} + x_{QD} = 27 \text{ or } \sum x_{Qj} = 27$	A1
$x_{RA} + x_{RB} + x_{RC} + x_{RD} = 28 \text{ or } \sum x_{Rj} = 28$ $x_{PA} + x_{QA} + x_{RA} = 18 \text{ or } \sum x_{iA} = 18$ $x_{PB} + x_{QB} + x_{RB} = 16 \text{ or } \sum x_{iB} = 16$ $x_{PC} + x_{QC} + x_{RC} = 20 \text{ or } \sum x_{iC} = 20$	A1
$x_{PD} + x_{QD} + x_{RD} = 26 \text{ or } \sum x_{iD} = 26$ $x_{ij} \geq 0$	A1
	7 marks

Notes

1B1: Variables defined correctly – withhold this mark if definition of x_{ij} or the elements of sets i and j are inconsistent with their later use in the objective function and constraints. Penalise poor variable choice, (AP etc.) here.

2B1: Minimise + an attempt at an objective with at least 5 correct terms.

3B1: Objective function correct (minimised not required for this mark).

1M1: At least 3 ‘correct’ constraints listed with **unit** coefficients (accept = or any inequality for the M mark) – rhs values must be correct.

1A1: At least 3 correct constraints (accept consistent use of = or $\leq$ on at least 3).

2A1: At least 6 correct constraints (accept consistent use of = or $\leq$ on at least 6).

3A1: All 8 constraints correct (first seven constraints consistently either = or $\leq$ but final constraint must be $\geq$ 0).

2.

Mark scheme (a)

Scheme	Marks
The solution would otherwise be degenerate	B1
B3	B1
	(2)

Notes

a1B1: 'degenerate' or an argument based on $n + m - 1$ required (values do not need to be substituted) – not just 'need 7 values', 'there are only 6 entries', 'so we can do a stepping-stone method' or 'demand and supply has been met'

a2B1: CAO (B3) – could be seen in a diagram (but must be clear)

Mark scheme (b)

Scheme	Marks																									
<table><tr><td></td><td>24 (1)</td><td>27 (2)</td><td>19 (3)</td><td>21 (4)</td></tr><tr><td>0 (A)</td><td>X</td><td>5</td><td>2</td><td>13</td></tr><tr><td>4 (B)</td><td>X</td><td>X</td><td>6</td><td>12</td></tr><tr><td>14 (C)</td><td>-13</td><td>X</td><td>X</td><td>X</td></tr><tr><td>15 (D)</td><td>-16</td><td>-10</td><td>-3</td><td>X</td></tr></table>		24 (1)	27 (2)	19 (3)	21 (4)	0 (A)	X	5	2	13	4 (B)	X	X	6	12	14 (C)	-13	X	X	X	15 (D)	-16	-10	-3	X	M1 A1 A1
	24 (1)	27 (2)	19 (3)	21 (4)																						
0 (A)	X	5	2	13																						
4 (B)	X	X	6	12																						
14 (C)	-13	X	X	X																						
15 (D)	-16	-10	-3	X																						
	(3)																									

Notes

b1M1: Finding 8 shadow costs and 9 IIs

b1A1: CAO for shadow costs [Alt: A(24), B(28), C(38), D(39), 1(0), 2(3), 3(-5), 4(-3)]

b2A1: CAO for improvement indices – must not be stated as part of a table of costs unless clearly differentiated from these costs e.g. circled, underlined, etc.

Mark scheme (c)

Scheme	Marks
Route is e.g. D1 – B1 – B2 – C2 – C4 – D4	M1
Entering cell is D1, Exiting cell is C2	A1
	(2)
	7 marks

Notes

c1M1: A valid route (possibly drawn so need not be explicitly stated), their most negative II chosen, only one empty square, θ 's balanced

c1A1: Correct stepping-stone route stated or clearly shown on a diagram and CAO for entering and exiting cells

SC1: Those candidates who have a 0 in B3 can score (b) M1A0A0 (c) M1A1 – the M in (b) is for finding 8 shadow costs and 9 IIs. The M mark in (c) is for the correct route D1 – B1 – B3 – C3 – C4 – D4 and the A mark is for the correct entering cell (D1) and correct exiting cell (B1)

SC2: Those candidates who have a 0 in C2 for (b) but then move it to B3 for (c) can score full marks in (b) and then M1 only in (c) for a valid balancing route

3.

Mark scheme (a)

Scheme	Marks
Adds a column of four zeros and 10.	B1
	(1)

Notes

1B1: cao

Mark scheme (b)

Scheme	Marks
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						M1 A1 M1 A1
Shadow costs		31	42	47	9	
		A	B	C	D	
0	1	x	-13	-15	-9	
-9	2	x	x	-11	0	
-15	3	9	x	x	6	
-9	4	1	-7	x	x	
						(4)

Notes

The shadow costs and the missing improvement indices are shown in bold.

1M1: Finding all 8 shadow costs

1A1: cao

2M1: Finding missing four improvement indices – no extra zeros

2A1: cao

Mark scheme (c)

Scheme	Marks																									
<table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td>1</td><td>$20 - \theta$</td><td></td><td>θ</td><td></td></tr><tr><td>2</td><td>$15 + \theta$</td><td>$7 - \theta$</td><td></td><td></td></tr><tr><td>3</td><td></td><td>$18 + \theta$</td><td>$2 - \theta$</td><td></td></tr><tr><td>4</td><td></td><td></td><td>28</td><td>10</td></tr></table> Entering cell: 1C Exiting cell: 3C		A	B	C	D	1	$20 - \theta$		θ		2	$15 + \theta$	$7 - \theta$			3		$18 + \theta$	$2 - \theta$		4			28	10	M1 A1ft
	A	B	C	D																						
1	$20 - \theta$		θ																							
2	$15 + \theta$	$7 - \theta$																								
3		$18 + \theta$	$2 - \theta$																							
4			28	10																						
<table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td>1</td><td>18</td><td></td><td>2</td><td></td></tr><tr><td>2</td><td>17</td><td>5</td><td></td><td></td></tr><tr><td>3</td><td></td><td>20</td><td></td><td></td></tr></table>		A	B	C	D	1	18		2		2	17	5			3		20			DM1 A1					
	A	B	C	D																						
1	18		2																							
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Notes

1M1: A valid route, their most negative II chosen, only one empty square used, θ 's balance.

1A1ft: consistent; their entering and exiting cells **stated clearly**

2DM1: An **improved** solution 7 entries only, (so must now be using one of my negative indices as the entering cell). Must ft from their valid route.

2A1: cao