

QQQ – Decision 2 – Chapter 1 – Transportation Problems (Version 1)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1. The table below shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to three demand points, P, Q and R. It also shows the stock held at each supply point and the number of units required at each demand point.

A minimum cost solution is required.

	P	Q	R	Supply
A	27	23	25	35
B	29	30	28	41
C	29	33	26	29
D	32	34	36	45
Demand	57	31	62	

- (a) Use the north-west corner method to obtain an initial solution to this transportation problem. (1)
- (b) Perform one iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by showing the route and stating the
- shadow costs
 - improvement indices
 - entering cell and exiting cell
- (4)
- (c) Formulate the transportation problem as a linear programming problem. You must define your decision variables and make the objective function and constraints clear. (5)

(Total 10 marks)

2. The table below shows the cost, in pounds, of transporting one unit of stock from each of four supply points, E, F, G and H, to three sales points, A, B and C. It also shows the stock held at each supply point and the amount required at each sales point.
A minimum cost solution is required.

	A	B	C	Supply
E	23	28	22	21
F	26	19	29	32
G	29	24	20	29
H	24	26	19	23
Demand	45	19	23	

- (a) Explain why it is necessary to add a dummy demand point. (1)
- (b) On Table 1 in the answer book, insert appropriate values in the dummy demand column, D. (1)

	A	B	C	D	Supply
E	23	28	22		21
F	26	19	29		32
G	29	24	20		29
H	24	26	19		23
Demand	45	19	23		

Table 1

After finding an initial feasible solution and applying one iteration of the stepping-stone method, the table becomes

	A	B	C	D
E	21			
F	19	13		
G		6	23	
H	5			18

(c) Starting with GD as the next entering cell, perform two further iterations of the stepping-stone method to obtain an improved solution. You must make your method clear by showing your routes and stating the

- shadow costs
- improvement indices
- entering and exiting cells

(6)

(d) State the cost of the solution found in (c). **(1)**

(e) Determine whether the solution obtained in (c) is optimal, giving a reason for your answer. **(3)**

(Total 12 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO																				
<table><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>A</td><td>35</td><td></td><td></td></tr><tr><td>B</td><td>22</td><td>19</td><td></td></tr><tr><td>C</td><td></td><td>12</td><td>17</td></tr><tr><td>D</td><td></td><td></td><td>45</td></tr></table>		P	Q	R	A	35			B	22	19		C		12	17	D			45	B1	1.1b
	P	Q	R																			
A	35																					
B	22	19																				
C		12	17																			
D			45																			
	(1)																					

Notes
B1: CAO for north-west corner method

Mark scheme (b)

Scheme	Marks	AO																														
<table><tr><td></td><td></td><td>27</td><td>28</td><td>21</td></tr><tr><td></td><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>0</td><td>A</td><td>X</td><td>−5</td><td>4</td></tr><tr><td>2</td><td>B</td><td>X</td><td>X</td><td>5</td></tr><tr><td>5</td><td>C</td><td>−3</td><td>X</td><td>X</td></tr><tr><td>15</td><td>D</td><td>−10</td><td>−9</td><td>X</td></tr></table>			27	28	21			P	Q	R	0	A	X	−5	4	2	B	X	X	5	5	C	−3	X	X	15	D	−10	−9	X	M1 A1	2.1 1.1b
		27	28	21																												
		P	Q	R																												
0	A	X	−5	4																												
2	B	X	X	5																												
5	C	−3	X	X																												
15	D	−10	−9	X																												
<table><tr><td></td><td>P</td><td>Q</td><td>R</td></tr><tr><td>A</td><td></td><td></td><td></td></tr><tr><td>B</td><td>$22 - \theta$</td><td>$19 + \theta$</td><td></td></tr></table>		P	Q	R	A				B	$22 - \theta$	$19 + \theta$		M1 A1	1.1b 2.2a																		
	P	Q	R																													
A																																
B	$22 - \theta$	$19 + \theta$																														

C		$12 - \theta$	$17 + \theta$
D	θ		$45 - \theta$

	P	Q	R
A	35		
B	10	31	
C			29
D	12		33

Entering cell is DP and exiting cell is CQ

(4)

Notes

M1: Finding 7 shadow costs and 6 improvement indices

A1: Shadow costs and II correct (Alternative SC rows 27, 29, 32, 42 columns 0, 1, -6)

M1: A valid route, their most negative II chosen, only one empty square used, θ 's balance

A1: CSO (for (b)) so all previous marks in this part must have been awarded – including exiting and entering cells stated correctly. Improved solution must be 6 numbers only with no additional 0 in CQ

Mark scheme (c)

Scheme	Marks	A0
Let x_{ij} be the number of units (of stock) transported from (supply point) i to (demand point) j	B1	3.3
where $i \in \{A, B, C, D\}$ and $j \in \{P, Q, R\}$ ($x_{ij} \geq 0$)	B1	2.5
Minimise $27x_{AP} + 23x_{AQ} + 25x_{AR} + 29x_{BP} + 30x_{BQ} + 28x_{BR}$ $+ 29x_{CP} + 33x_{CQ} + 26x_{CR} + 32x_{DP} + 34x_{DQ} + 36x_{DR}$	B1	3.3
$\sum x_{Aj} \leq 35$, $\sum x_{Bj} \leq 41$, $\sum x_{Cj} \leq 29$, $\sum x_{Dj} \leq 45$ accept =	B1	3.3
$\sum x_{iP} \geq 57$, $\sum x_{iQ} \geq 31$, $\sum x_{iR} \geq 62$ accept =	B1	3.3
	(5)	

	(10 marks)	
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Notes

(c) Check all suffixes carefully for accuracy and consistency

B1: Correct definition of x_{ij} must clearly state that this is the **number** of units transported

B1: Correctly defining the set of values that i and j can take

B1: 'Minimise' + correct objective function

B1: Correct supply constraints with unit coefficients

(allow 'equals' or written out in full e.g. $x_{AP} + x_{AQ} + x_{AR} \leq 35$)

B1: Correct demand constraints with unit coefficients

(allow 'equals' or written out in full e.g. $x_{AP} + x_{BP} + x_{CP} + x_{DP} \geq 57$)

2.

Mark scheme (a)

Scheme	Marks	AO
(total) demand $\neq$ (total) supply	B1	1.2
	(1)	

Notes

B1: CAO (or to make demand = supply or because (total) supply > (total) demand (oe))

Accept e.g. A dummy demand of 18 is needed to meet supply

or there is a total of 105 supply but only 87 demand

Mark scheme (b)

Scheme	Marks	AO
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	A	B	C	Dummy	Supply	B1	1.1b
E	23	28	22	0	21		
F	26	19	29	0	32		
G	29	24	20	0	29		
H	24	26	19	0	23		
Demand	45	19	23	18			
						(1)	

Notes

B1: CAO Check 18 in demand row

Mark scheme (c)

Scheme	Marks	AO																																																		
<table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td>E</td><td></td><td></td><td></td><td></td></tr><tr><td>F</td><td>$19 - \theta$</td><td>$13 + \theta$</td><td></td><td></td></tr><tr><td>G</td><td></td><td>$6 - \theta$</td><td></td><td>θ</td></tr><tr><td>H</td><td>$5 + \theta$</td><td></td><td></td><td>$18 - \theta$</td></tr></table> <table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td>E</td><td>21</td><td></td><td></td><td></td></tr><tr><td>F</td><td>13</td><td>19</td><td></td><td></td></tr><tr><td>G</td><td></td><td></td><td>23</td><td>6</td></tr><tr><td>H</td><td>11</td><td></td><td></td><td>12</td></tr></table> Exiting cell is GB		A	B	C	D	E					F	$19 - \theta$	$13 + \theta$			G		$6 - \theta$		θ	H	$5 + \theta$			$18 - \theta$		A	B	C	D	E	21				F	13	19			G			23	6	H	11			12	M1 A1	2.1 1.1b
	A	B	C	D																																																
E																																																				
F	$19 - \theta$	$13 + \theta$																																																		
G		$6 - \theta$		θ																																																
H	$5 + \theta$			$18 - \theta$																																																
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E	21																																																			
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G			23	6																																																
H	11			12																																																
<table><tr><td></td><td></td><td>23</td><td>16</td><td>19</td><td>-1</td></tr><tr><td></td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr></table>			23	16	19	-1			A	B	C	D	M1 A1	1.1b 1.1b																																						
		23	16	19	-1																																															
		A	B	C	D																																															

0	E	X	12	3	1
3	F	X	X	7	-2
1	G	5	7	X	X
1	H	X	9	-1	X

	A	B	C	D
E				
F	$13 - \theta$			θ
G				
H	$11 + \theta$			$12 - \theta$

	A	B	C	D
E	21			
F	1	19		12
G			23	6
H	23			

Entering cell is FD and exiting cell is HD

M1
A1

1.1b
2.2a

(6)

Notes

M1: A valid route, only one empty square (GD) used, θ s balance

A1: Correct route, up to an improved solution (seven numbers no zeros)

M1: Finding 8 shadow costs and 9 improvement indices

A1: Shadow costs and II correct (alternatives columns 0 -7 -4 -24 rows 23 26 24 24)

M1: A valid route, their most negative II chosen, only one empty square used, θ s balance

A1: CSO (for part (c)) so all previous marks in this part must have been awarded – including exiting cells (GB and HD) and entering cell (FD) stated correctly (seven numbers no zeros)

Mark scheme (d)

Scheme	Marks	AO
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(£)1882	B1	1.1b
	(1)	

Notes

B1: CAO

Mark scheme (e)

Scheme	Marks	AO																																				
<table><tr><td></td><td></td><td>23</td><td>16</td><td>17</td><td>−3</td></tr><tr><td></td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td>0</td><td>E</td><td>X</td><td>12</td><td>5</td><td>3</td></tr><tr><td>3</td><td>F</td><td>X</td><td>X</td><td>9</td><td>X</td></tr><tr><td>3</td><td>G</td><td>3</td><td>5</td><td>X</td><td>X</td></tr><tr><td>1</td><td>H</td><td>X</td><td>9</td><td>1</td><td>2</td></tr></table>			23	16	17	−3			A	B	C	D	0	E	X	12	5	3	3	F	X	X	9	X	3	G	3	5	X	X	1	H	X	9	1	2	M1 A1	2.1 1.1b
		23	16	17	−3																																	
		A	B	C	D																																	
0	E	X	12	5	3																																	
3	F	X	X	9	X																																	
3	G	3	5	X	X																																	
1	H	X	9	1	2																																	
All II are non-negative so solution is optimal	A1	2.4																																				
	(3)																																					
	(12 marks)																																					

Notes

M1: Finding 8 shadow costs and all 9 improvement indices (or 8 SC and at least 1 negative II)

A1: CAO for shadow costs and the 9 improvement indices (alternatives columns 0 -7 -6 -26 rows 23 26 26 24)

A1: CSO (for part (e)) + reason + optimal) accept positive instead of non-negative