

QQQ – Decision 1 – Chapter 7 – The Simplex Algorithm (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. A linear programming problem in x , y and z is to be solved using the big-M method. The initial tableau is shown below.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	2	3	4	1	0	0	0	0	13
a_1	1	-2	2	0	-1	0	1	0	8
a_2	3	0	-4	0	0	-1	0	1	12
P	$2 - 4M$	$-3 + 2M$	$-1 + 2M$	0	M	M	0	0	$-20M$

(a) Using the information in the above tableau, formulate the linear programming problem. You should

- list each of the constraints as an inequality
- state the two possible objectives

(4)

(b) Obtain the most efficient pivot for a first iteration of the big-M method. You must give reasons for your answer.

(2)

(Total 6 marks)

2. Susie is preparing for a triathlon event that is taking place next month. A triathlon involves three activities: swimming, cycling and running.

Susie decides that in her training next week she should

- maximise the total time spent cycling and running
- train for at most 39 hours
- spend at least 40% of her time swimming
- spend a total of at least 28 hours of her time swimming and running

Susie needs to determine how long she should spend next week training for each activity. Let

- x represent the number of hours swimming
- y represent the number of hours cycling
- z represent the number of hours running

(a) Formulate the information above as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients. (5)

Susie decides to solve this linear programming problem by using the two-stage Simplex method.

(b) Set up an initial tableau for solving this problem using the two-stage Simplex method. As part of your solution you must show how

- the constraints have been made into equations using slack variables, exactly one surplus variable and exactly one artificial variable
- the rows for the two objective functions are formed

(6)

The following tableau T is obtained after one iteration of the second stage of the two-stage Simplex method.

b.v.	x	y	z	s_1	s_2	s_3	Value
y	0	1	0	1	0	1	11
s_2	0	0	5	-2	1	-5	62
x	1	0	1	0	0	-1	28
P	0	0	-1	1	0	1	11

(c) Obtain a suitable pivot for a second iteration. You must give reasons for your answer. (2)

- (d) Starting from tableau T , solve the linear programming problem by performing one further iteration of the second stage of the two-stage Simplex method. You should make your method clear by stating the row operations you use. (5)

(Total 18 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks	AO
Constraints:	B1	3.4
$2x + 3y + 4z \leq 13$	B1	2.5
$x - 2y + 2z \geq 8$		
$3x - 4z \geq 12$		
$(x, y, z \geq 0)$		
Objective functions:	M1	3.1a
Maximise $-2x + 3y + z$	A1	2.2a
Minimise $2x - 3y - z$		
	(4)	

Notes

(a) B1: One correct non-trivial inequality (allow strict inequality provided direction of inequality sign is correct) – equations with slack variables etc. scores no marks unless replaced with correct inequalities

B1: All three non-trivial inequalities correct

M1: Either expression stated correctly (allow equal to (or an inequality with) any letter e.g.

$P = -2x + 3y + z$ but not equal to a value e.g. $= 0$) – ignore any mention of maximum/minimum for this mark

A1: Both expressions correct including max/min correctly matched with each expression (allow equal to any letter only) – do not isw if they continue and place their expression(s) equal to a value(s)

Mark scheme (b)

Scheme	Marks	AO
(Because M is big) the only negative in the objective row is the $2 - 4M$ so the pivot is from the x -column	B1	2.4
The 3 in the a_2 row is the pivot as $\frac{12}{3}$ is less than both $\frac{8}{1}$ and $\frac{13}{2}$	B1	2.2a
	(2)	

(6 marks)

Notes

(b) B1: Correct reasoning that the pivot is a value from the x -column – as a minimum must state that the $2 - 4M$ is the only negative (condone most negative) in the objective row (allow profit row or P row, condone 'bottom row')
B1: Correct justification of why the 3 in the a_2 row or the 3 in the x column is the pivot – so **must** state the correct pivot in a clear unambiguous way (so just saying the pivot is 'the 3' is B0) **and** comparing or stating that $\frac{12}{3}$ or 4 is less than/least positive for both $\frac{8}{1}$ or 8 **and** $\frac{13}{2}$ or 6.5 – **must** see all three values so do check the table for possibly stating the θ values there. However, just stating that the 3 is the pivot because it is the smallest θ value (without seeing anywhere these θ values) is B0

2.

Mark scheme (a)

Scheme	Marks	AO
$x + y + z \leq 39$	B1	3.3
$\frac{2}{5}(x + y + z) \leq x \quad (\Rightarrow -3x + 2y + 2z \leq 0)$	M1 A1	3.3 1.1b
$x + z \geq 28$	B1	1.1b
Maximise $P = y + z \quad (\Rightarrow P - y - z = 0)$	B1	3.3
	(5)	

Notes

(a) B1: cao ($x + y + z \leq 39$)

M1: $\frac{2}{5}(x + y + z) \square x$ where $\square$ is any inequality or equals

A1: cao

B1: cao ($x + z \geq 28$)

B1: Correct objective function ($P = y + z$) plus 'maximise' or 'max' but not 'maximum'

Mark scheme (b)

Scheme	Marks	AO
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$x + y + z \leq 39 \Rightarrow x + y + z + s_1 = 39$	M1	2.1																																																						
$-3x + 2y + 2z \leq 0 \Rightarrow -3x + 2y + 2z + s_2 = 0$	A1	1.1b																																																						
$x + z \geq 28 \Rightarrow x + z - s_3 + a_1 = 28$	B1	2.5																																																						
$I = -a_1 \Rightarrow I - x - z + s_3 = -28$	M1	2.1																																																						
e.g. <table><tr><th>b.v</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>a_1</th><th>Value</th></tr><tr><td>s_1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>39</td></tr><tr><td>s_2</td><td>-3</td><td>2</td><td>2</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>a_1</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td><td>-1</td><td>1</td><td>28</td></tr><tr><td>P</td><td>0</td><td>-1</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>I</td><td>-1</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>-28</td></tr></table>	b.v	x	y	z	s_1	s_2	s_3	a_1	Value	s_1	1	1	1	1	0	0	0	39	s_2	-3	2	2	0	1	0	0	0	a_1	1	0	1	0	0	-1	1	28	P	0	-1	-1	0	0	0	0	0	I	-1	0	-1	0	0	1	0	-28	M1 A1	3.3 2.2a
b.v	x	y	z	s_1	s_2	s_3	a_1	Value																																																
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a_1	1	0	1	0	0	-1	1	28																																																
P	0	-1	-1	0	0	0	0	0																																																
I	-1	0	-1	0	0	1	0	-28																																																
	(6)																																																							

Notes

(b) M1: One $\leq$ constraint re-formulated as an equation using slack variables – dependent on either the first **B** mark in **(a)** or the **M** mark in **(a)**

A1: cao (both $\leq$ constraints)

B1: $\geq$ constraint re-formulated as an equation using one surplus and one artificial variable

M1: Formulates second objective with $I = -a_1$ and their expression for a_1

M1: Setting up the initial tableau – all five rows complete with two correct rows (but ignore b.v. column for this mark)

A1: cao (any equivalent correct form)

Mark scheme (c)

Scheme	Marks	AO
The only negative in the objective row is the -1 so the pivot is from the z -column	B1	2.4
The 5 in the s_2 row is the pivot because $\frac{62}{5}$ is less than $\frac{28}{1}$	B1	2.2a

(2)

Notes

(c) B1: Correct reasoning that the pivot is a value from the z -column – condone any mention of negative value in P row

B1: Correct justification of why the 5 in the s_2 row is the next pivot – so must compare or state that 12.4 is less than 28 (not sufficient to just say that 12.4 (oe) is the least)

Mark scheme (d)

Scheme									Marks	AO
b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops	B1 M1 A1 A1	1.1b 2.1 1.1b 1.1b
y	0	1	0	1	0	1	11	R1		
z	0	0	1	$-\frac{2}{5}$	$\frac{1}{5}$	-1	$\frac{62}{5}$	$\frac{1}{5}R2$		
x	1	0	0	$\frac{2}{5}$	$-\frac{1}{5}$	0	$\frac{78}{5}$	$R3 - R2$		
P	0	0	0	$\frac{3}{5}$	$\frac{1}{5}$	0	$\frac{117}{5}$	$R4 + R2$		
Spend 15.6 hours swimming, 11 hours cycling and 12.4 hours running									A1	3.2a
									(5)	
									(18 marks)	

Notes

(d) B1: Pivot row correct including change of b.v.

M1: All values in one of the non-pivot rows correct or one of the non zero and one columns (s_1 , s_2 or value) correct (from their choice of pivot)

A1: Row operations used correctly at least twice, i.e. two of the non zero and one columns (s_1 , s_2 or value)

A1: For all values and row operations correctly stated – do not penalise lack of correct b.v. in pivot row twice. Condone blank Row Ops in the first row only

A1: Correct allocation of training times – must be in context (so not just in terms of x , y and z)