

QQQ – Decision 1 – Chapter 7 – The Simplex Algorithm (Version 1)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. The tableau below is the initial tableau for a linear programming problem in x , y and z . The objective is to maximise the profit, P .

Basic variable	x	y	z	r	s	t	Value
r	2	-4	1	1	0	0	15
s	4	2	-8	0	1	0	20
t	1	-1	4	0	0	1	8
P	-3	2	7	0	0	0	0

- (a) Perform one iteration of the Simplex algorithm to obtain a new tableau, T. State the row operations you use. (5)
- (b) Write down the profit equation given by T and state the current values of the slack variables. (2)

(Total 7 marks)

2. A maximisation linear programming problem in x , y and z is to be solved using the two-stage simplex method.

The partially completed initial tableau is shown below.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	1	2	3	1	0	0	0	0	45
a_1	3	2	0	0	-1	0	1	0	9
a_2	-1	0	4	0	0	-1	0	1	4
P	-2	-1	-3	0	0	0	0	0	0
A									

- (a) Using the information in the above tableau, formulate the linear programming problem. State the objective and list the constraints as inequalities. (4)
- (b) Complete the bottom row of Table 1 in the answer book. You should make your method and working clear. (2)

[Table 1 in the answer book is a copy of the partially completed initial tableau above.]

The following tableau is obtained after two iterations of the first stage of the two-stage simplex method.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	0	$\frac{5}{6}$	0	1	$\frac{7}{12}$	$\frac{3}{4}$	$-\frac{7}{12}$	$-\frac{3}{4}$	$\frac{147}{4}$
x	1	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	3
z	0	$\frac{1}{6}$	1	0	$-\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{7}{4}$
P	0	$\frac{5}{6}$	0	0	$-\frac{11}{12}$	$-\frac{3}{4}$	$\frac{11}{12}$	$\frac{3}{4}$	$\frac{45}{4}$
A	0	0	0	0	0	0	1	1	0

- (c) (i) Explain how the above tableau shows that a basic feasible solution has been found for the original linear programming problem.
- (ii) Write down the basic feasible solution for the second stage. (3)
- (d) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method, to obtain a new

tableau, T . Make your method clear by stating the row operations you use. **(5)**

(e) (i) Explain, using T , whether or not an optimal solution to the original linear programming problem has been found.

(ii) Write down the value of the objective function.

(iii) State the values of the basic variables. **(3)**

(Total 17 marks)

Solutions

1.

Mark scheme (a)

Scheme									Marks	
	b.v.	x	y	z	r	s	t	Value	Row ops	M1
	r	0	-5	5	1	$-\frac{1}{2}$	0	5	$R_1 - 2R_2$	A1
	x	1	$\frac{1}{2}$	-2	0	$\frac{1}{4}$	0	5	$R_2 \div 4$	M1
	t	0	$-\frac{3}{2}$	6	0	$-\frac{1}{4}$	1	3	$R_3 - R_2$	A1ft
	P	0	$\frac{7}{2}$	1	0	$\frac{3}{4}$	0	15	$R_4 + 3R_2$	A1
									(5)	

Notes

a1M1: Correct pivot located (4 in column x), attempt to divide row

a1A1: Pivot row correct **including change of b.v.**

a2M1: All values in one of the non-pivot rows correct **or** one of the non zero and one columns (y, z, s or value) correct following through their choice of pivot from column x

a2A1ft: Row operations used correctly at least twice, i.e. **two** of the non zero and one columns (y, z, s or value) correct following through their choice of pivot from column x

a3A1: CAO – no follow through – all values and row operations correctly stated – allow if row operations given in terms of old row 2 – **ignore b.v. column for this mark**

Pivoting on the 1 in the x -column

b.v.	x	y	z	r	S	t	V
r	0	-2	-7	1	0	-2	-1
s	0	6	-24	0	1	-4	-12
x	1	-1	4	0	0	1	8
P	0	-1	19	0	0	3	24

Pivoting on the 2 in the x -column

b.v.	x	y	z	r	s	t	V
x	1	-2	0.5	0.5	0	0	7.5
s	0	10	-10	-2	1	0	-10
t	0	1	3.5	-0.5	0	1	0.5
P	0	-4	8.5	1.5	0	0	22.5

Mark scheme (b)

Scheme	Marks
$P + \frac{7}{2}y + z + \frac{3}{4}s = 15$	B1ft
$r = 5, s = 0, t = 3$	B1
	(2)
	7 marks

Notes

b1B1ft: Follow their profit equation from (a) dependent on scoring **both** M marks in (a)

b2B1: CAO (no follow through) for slack variables ($r = 5, s = 0, t = 3$)

2.

Mark scheme (a)

Scheme	Marks	AO
Maximise ($P =$) $2x + y + 3z$	B1	3.4
$x + 2y + 3z \leq 45$	B1	3.4
$3x + 2y \geq 9$	B1	3.4
$-x + 4z \geq 4$	B1	1.1b
	(4)	

Notes

(a) **B1:** CAO – including maximise (or max)

B1: CAO (oe)

B1: CAO (oe)

B1: CAO (oe)

Mark scheme (b)

Scheme	Marks	AO										
$A = -(a_1 + a_2) \Rightarrow -(9 - 3x - 2y + s_2 + 4 + x - 4z + s_3)$	M1	2.1										
$A - 2x - 2y - 4z + s_2 + s_3 = -13$ therefore bottom row of the table is <table><tr><td>A</td><td>-2</td><td>-2</td><td>-4</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>-13</td></tr></table>	A	-2	-2	-4	0	1	1	0	0	-13	A1	2.2a
A	-2	-2	-4	0	1	1	0	0	-13			
	(2)											

Notes

(b) M1: Setting up the new objective and substituting for a_1 and a_2

A1: Correct values substituted into Table 1

Mark scheme (c)

Scheme	Marks	AO
(i) In the given tableau the value of the objective A is equal to zero indicating that a basic feasible solution has been found	B1	2.4
(ii) $x = 3$, $y = 0$, $z = \frac{7}{4}$, $s_1 = \frac{147}{4}$, $s_2 = s_3 = 0$	B1 B1	3.4 1.1b
	(3)	

Notes

(c) B1: CAO – mention that $A = 0$

B1: At least three values stated correctly

B1: All six values correct (ignore values stated for a_1 , a_2 and P)

Mark scheme (d)

Scheme	Marks	AO
--------	-------	----

									M1	2.1
									A1	1.1b
									M1	1.1b
									A1	1.1b
									A1	1.1b
									(5)	

Notes

(d) **M1**: Correct pivot located, attempt to divide row

A1: Pivot row correct including change of b.v.

M1: All values in one of the non-pivot rows correct **or** one of the non zero and one columns (y , s_1 , s_3 or value) correct following through their choice of pivot from column s_2 or s_3

A1: Row operations used correctly at least twice, i.e. **two** of the non zero and one columns (y , s_1 , s_3 or value) correct

A1: CAO all values and row operations correctly stated – allow if row operations given in terms of old row 1 – **ignore b.v. column for this mark**

Mark scheme (e)

Scheme	Marks	A0
(i) Yes, an optimal solution has been found as there are no negative values in the objective (P) row	B1	2.4
(ii) $P = 69$	B1ft	3.4
(iii) $s_2 = 63$, $x = 24$, $z = 7$	B1ft	3.4
	(3)	
	(17 marks)	

Notes

(e)(i) **B1**: Correct reasoning of why solution is optimal or using $P = 69 - \frac{15}{7}y - \frac{11}{7}s_1 - \frac{3}{7}s_3$ and mentioning increasing y , s_1 , s_3 would decrease P (oe)

(e)(ii) **B1ft**: their value of P – dependent on both M marks in (d)

(e)(iii) **B1ft:** their values of the basic variables **only** – dependent on both M marks in (d)