

QQQ – Decision 1 – Chapter 6 – Linear Programming (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. A manufacturer of frozen yoghurt is going to exhibit at a trade fair. He will take two types of frozen yoghurt, Banana Blast and Strawberry Scream.

He will take a total of at least 1000 litres of yoghurt.

He wants at least 25% of the yoghurt to be Banana Blast. He also wants there to be at most half as much Banana Blast as Strawberry Scream.

Each litre of Banana Blast costs £3 to produce and each litre of Strawberry Scream costs £2 to produce. The manufacturer wants to minimise his costs.

Let x represent the number of litres of Banana Blast and y represent the number of litres of Strawberry Scream.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(6)

(Total 6 marks)

2.

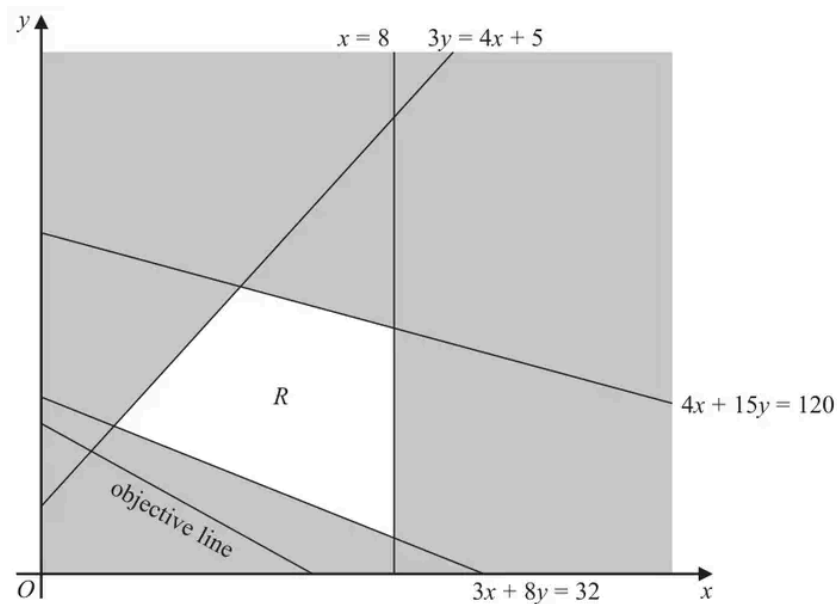


Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y .

The unshaded area, including its boundaries, forms the feasible region, R .

An objective line has been drawn and labelled on the graph.

(a) State the inequalities that define the feasible region. **(2)**

The maximum value of the objective function is $\frac{160}{3}$

The minimum value of the objective function is $\frac{883}{41}$

(b) Determine the objective function, showing your working clearly. **(5)**

(Total 7 marks)

3. The manager of a factory is planning the production schedule for the next three weeks for a range of cabinets. The following constraints apply to the production schedule.

- The total number of cabinets produced in week 3 cannot be fewer than the total number produced in weeks 1 and 2
- At most twice as many cabinets must be produced in week 3 as in week 2
- The number of cabinets produced in weeks 2 and 3 must, in total, be at most 125

The production cost for each cabinet produced in weeks 1, 2 and 3 is £250, £275 and £200 respectively.

The factory manager decides to formulate a linear programming problem to find a production schedule that minimises the total cost of production.

The objective is to minimise $250x + 275y + 200z$

(a) Explain what the variables x , y and z represent. **(1)**

(b) Write down the constraints of the linear programming problem in terms of x , y and z . **(2)**

Due to demand, exactly 150 cabinets must be produced during these three weeks. This reduces the constraints to

$$\begin{aligned}x + y &\leq 75 \\x + 3y &\geq 150 \\x &\geq 25 \\y &\geq 0\end{aligned}$$

which are shown in Diagram 1 in the answer book.

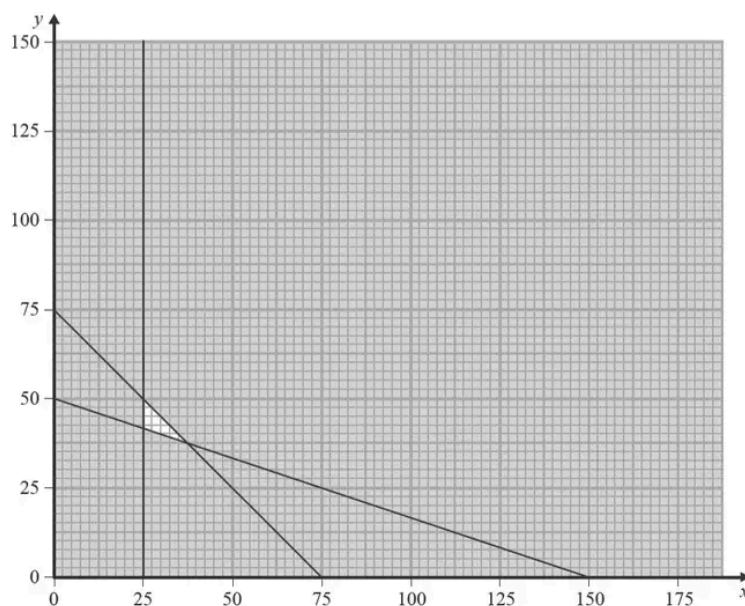


Diagram 1

Given that the manager does not want any cabinets left unfinished at the end of a week,

(c) (i) use a graphical approach to solve the linear programming problem and hence determine the production schedule which minimises the cost of production. You should make your method and working clear.

(ii) Find the minimum total cost of the production schedule. **(8)**

(Total 11 marks)

Solutions

1.

Scheme	Marks
Minimise $C = 3x + 2y$	B1
Subject to:	
$x + y \geq 1000$	B1
$\frac{1}{4}(x + y) \leq x$, simplifies to $y \leq 3x$	M1 A1
$2x \leq y$	M1 A1
$(x, y \geq 0)$	
	(6 marks)

Notes

1B1: CAO – expression correct and ‘minimise’.

2B1: CAO

1M1: Correct method – must see $\frac{1}{4}(x + y) \blacksquare x$ where $\blacksquare$ is any inequality or =. The bracket must be present or implied by later working.

1A1: CAO – simplified – answer must have integer coefficients.

2M1: Correct method – one of $2x \blacksquare y$ or $x \blacksquare 2y$ where $\blacksquare$ is any inequality or =.

2A1: CAO – answer must have integer coefficient.

2.

Mark scheme (a)

Scheme	Marks	AO
$x \leq 8, \quad 3x + 8y \geq 32, \quad 3y \leq 4x + 5, \quad 4x + 15y \leq 120$	B2, 1, 0	3.3 1.1b
	(2)	

Notes

B1: Any two correct inequalities. Condone strict inequalities

B1: All four correct inequalities (not strict)

Mark scheme (b)

Scheme	Marks	AO
Attempt to solve correct two equations to find either optimal vertex	M1	3.4
Coordinates of 'minimum' point is $\left(\frac{56}{41}, \frac{143}{41}\right)$	A1	1.1b
Coordinates of 'maximum' point is $\left(8, \frac{88}{15}\right)$	A1	1.1b
Setting up a pair of simultaneous equations using their two points and an objective function of the form $ax + by$ $56a + 143b = 883$ (If correct $15a + 11b = 100$ ^{oe})	dM1	3.1a
Objective function is $(P =) 3x + 5y$	A1	2.2a
	(5)	
	(7 marks)	

Notes

M1: Considering either of the following pairs of simultaneous equations:

$$3y = 4x + 5, 3x + 8y = 32 \text{ or } x = 8, 4x + 15y = 120$$

Must find at least one pair of coordinates from either of these two pairs (condone poor algebra in the solving of these equations)

A1: $\left(\frac{56}{41}, \frac{143}{41}\right)$ - must be seen exact at some point. They do not have to associate this with being the 'minimum'

A1: $\left(8, \frac{88}{15}\right)$ - must be seen exact at some point. They do not have to associate this with being the 'maximum'

dM1: Setting up a pair of linear simultaneous equations using their two points. For this mark they must be using their solution of $3y = 4x + 5, 3x + 8y = 32$ together with $\frac{883}{41}$ and their solution of

$$x = 8, 4x + 15y = 120 \text{ together with } \frac{160}{3}. \text{ Allow use of any two different variables for their pair of}$$

linear simultaneous equations. Look out for $\frac{56}{41}x + \frac{143}{41}y = \frac{883}{41}$ and $8x + \frac{88}{15}y = \frac{160}{3}$ which implies the first four marks

A1: cao – allow just the expression $3x + 5y$ but not any multiple or factor of this (but isw if correct expression is seen first). Allow equal to any other letter but not equal to a value, for example, $3x + 5y = 0$ is **A0**

3.

Mark scheme (a)

Scheme	Marks	A0
x is the number of cabinets produced in week 1, y is the number of cabinets produced in week 2 and z is the number of cabinets produced in week 3	B1	2.5
	(1)	

Notes

B1: Cao - must contain 'number of...' (oe e.g. 'amount of', 'quantity of',...) at least once

Mark scheme (b)

Scheme	Marks	A0
$x + y \leq z$	B1	3.3
$z \leq 2y$	B1	3.3
$y + z \leq 125$		
$(x, y, z \geq 0)$		
	(2)	

Notes

B1: Any one correct (accept strict inequalities)

B1: All three correct

Mark scheme (c)

Scheme	Marks	A0
(i) Objective is $P = 250x + 275y + 200(150 - x - y)$	M1	3.1a
$P = 50x + 75y (+ 30000)$	A1	1.1b
Objective line drawn or at least two vertices tested	M1	3.1a

Optimal point $\left(25, \frac{125}{3}\right)$	A1	1.1b
Consideration of integer coordinates around the optimal vertex	M1	1.1b
Correct integer coordinate (25, 42)	A1	1.1b
The production schedule is 25 cabinets in week 1, 42 cabinets in week 2 and 83 cabinets in week 3	B1	3.2a
(ii) Total cost of production is £34 400	B1	1.1b
	(8)	
	(11 marks)	

Notes

Note that the vertices of the FR are $\left(25, \frac{125}{3}\right), (25, 50), \left(\frac{75}{2}, \frac{75}{2}\right)$

(c)(i) M1: Attempt to derive new objective function in terms of x and y only by using $x + y + z = 150$
or attempt to calculate all three values of z using $x + y + z = 150$

A1: Cao for objective in terms of x and y only **or** all three correct z values $\left(\frac{250}{3}, 75, 75\right)$

M1: Objective line drawn consistent with their objective function (or its reciprocal) **or** testing two of the correct vertices (to at least 1 decimal place where applicable) in their objective function involving x and y only **or** testing two of the correct vertices (to at least 1 decimal place where applicable) in $250x + 275y + 200z$

A1: Correct optimal point $\left(25, \frac{125}{3}\right)$ **or** $\left(25, \frac{125}{3}, \frac{250}{3}\right)$ - accept 41.6 or 41.7 (or better) - so at least 1 decimal place (truncated or rounded) if not given exact

M1: Consideration of integer point(s) (e.g. (25, 41) etc.) around the optimal vertex - must have attempted point testing of the vertices of the feasible region or objective line

A1: Correct integer coordinate (25, 42) stated and either clear rejection of (26, 41) - by checking in $x + 3y \geq 150$ or testing of (27, 41) in a correct objective function

B1: Cao (in context - so not in terms of x, y and z)

(c)(ii) B1: Cao (£34,400) - condone lack of units