

QQQ – Decision 1 – Chapter 6 – Linear Programming (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. A firm is planning to produce two types of radio, type A and type B.

Market research suggests that, each week:

- At least 50 type A radios should be produced.
- The number of type A radios should be between 20% and 40% of the total number of radios produced.

Each type A radio requires 3 switches and each type B radio requires 2 switches. The firm can only buy 200 switches each week.

The profit on each type A radio is £15.

The profit on each type B radio is £12.

The firm wishes to maximise its weekly profit.

Formulate this situation as a linear programming problem, defining your variables.

(Total 7 marks)

2.

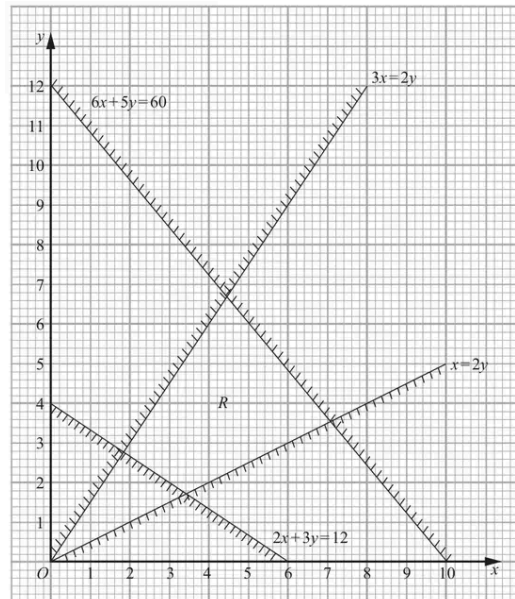


Figure 2

Figure 2 shows the constraints of a linear programming problem in x and y , where R is the feasible region.

- (a) Write down the inequalities that form region R . (2)

The objective is to maximise $3x + y$.

- (b) Find the optimal values of x and y . You must make your method clear. (4)

- (c) Obtain the optimal value of the objective function. (1)

Given that integer values of x and y are now required,

- (d) write down the optimal values of x and y . (1)

(Total 8 marks)

3.

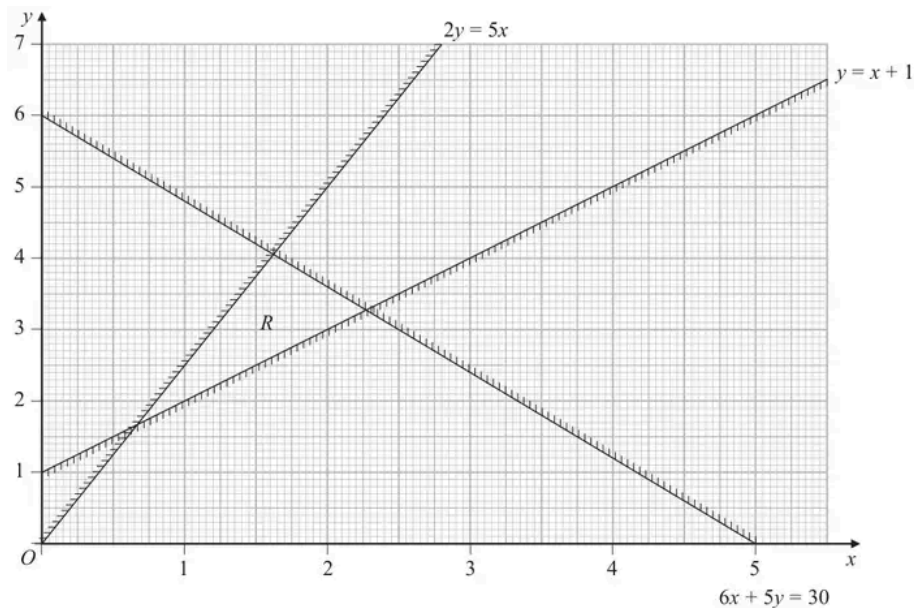


Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y , where R is the feasible region.

- (a) Write down the inequalities that define R . (2)

The objective is to maximise P , where $P = 3x + y$

- (b) Obtain the exact value of P at each of the three vertices of R and hence find the optimal vertex, V . (4)

The objective is changed to maximise Q , where $Q = 3x + ay$. Given that a is a constant and the optimal vertex is still V ,

- (c) find the range of possible values of a . (4)

(Total 10 marks)

Solutions

1.

Scheme	Marks
Let x be the number of type A radios and y be the number of type B radios.	B1
(Maximise P =) $15x + 12y$	B1
Subject to $x \geq 50$	B1
$\frac{1}{5}(x + y) < x$ (accept $\leq$) [$y < 4x$]	B1
$\frac{2}{5}(x + y) > x$ (accept $\geq$) [$2y > 3x$]	B1
$3x + 2y \leq 200$	B1
$y \geq 0$	B1
	(7 marks)

Notes

1B1 Defining x and y ; Must see 'number of'

2B1 CAO objective function $15x + 12y$

3B1 CAO $x \geq 50$

4B1 CAO o.e $\frac{1}{5}(x + y) < x \Rightarrow y < 4x$

5B1 CAO o.e $\frac{2}{5}(x + y) > x \Rightarrow 2y > 3x$ (corrected from the printed mark scheme: the notes print $\frac{2}{3}(x + y)$; the scheme above has $\frac{2}{5}(x + y)$, i.e. 40%)

6B1 CAO o.e $3x + 2y \leq 200$

7B1 CAO $y \geq 0$

2.

Mark scheme (a)

Scheme	Marks
$6x + 5y \leq 60$ $2x + 3y \geq 12$	B2,1,0

$3x \geq 2y$ $x \leq 2y$	
	(2)

Notes

(a)1B1 Any two inequalities correct, accept < and > here (but not = of course).

2B1 All four correct. Must be $\leq$ and $\geq$ here

Mark scheme (b)

Scheme	Marks
Drawing objective line{ (0,3) (1,0)} Testing at least 2 points	M1 A1
Calculating optimal point Testing at least 3 points	DM1
$\left(7\frac{1}{17}, 3\frac{9}{17}\right) = \left(\frac{120}{17}, \frac{60}{17}\right) \approx (7.06, 3.53)$	A1 awrt
	(4)

Notes

$$\left(3\frac{3}{7}, 1\frac{5}{7}\right) = \left(\frac{24}{7}, \frac{12}{7}\right) \approx (3.43, 1.71) \rightarrow 12$$

$$\left(1\frac{11}{13}, 2\frac{10}{13}\right) = \left(\frac{24}{13}, \frac{36}{13}\right) \approx (1.85, 2.77) \rightarrow 8.307692 \quad \left(8\frac{4}{13} = \frac{108}{13}\right)$$

$$\left(4\frac{4}{9}, 6\frac{2}{3}\right) = \left(\frac{40}{9}, \frac{20}{3}\right) \approx (4.44, 6.67) \rightarrow 20$$

$$\left(7\frac{1}{17}, 3\frac{9}{17}\right) = \left(\frac{120}{17}, \frac{60}{17}\right) \approx (7.06, 3.53) \rightarrow 24.705882 \quad \left(24\frac{12}{17} = \frac{420}{17}\right)$$

(b)1M1 Drawing objective line or its reciprocal OR testing two vertices in the feasible region (see list above) points correct to 1 dp.

1A1 Correct objective line OR two points correctly tested (1 dp ok)

2DM1 Calculating optimal point either answer to 2 dp or better or using S.E's (correct 2 equations for their point + attempt to eliminate one variable.); OR Testing three points correctly and optimal one to 2dp.

2A1 CAO 2 dp or better.

Mark scheme (c)

Scheme	Marks
$24\frac{12}{17} = \frac{420}{17} \approx 24.7$ (awrt)	B1
	(1)

Notes

(c) **B1** CAO

(Corrected from the printed mark scheme: the scheme prints $\frac{240}{17}$; $24\frac{12}{17} = \frac{420}{17}$, as in the list of points above.)

Mark scheme (d)

Scheme	Marks
(6,4)	B1
	(1)
	(8 marks)

Notes

(d) **B1** CAO not (4,6).

3.

Mark scheme (a)

Scheme	Marks	AO
$2y \leq 5x, y \geq x + 1, 6x + 5y \leq 30$	B2,1,0	1.1b 2.5
	(2)	

Notes

(a) **B1**: Any two correct (accept strict inequalities) – accept equivalent inequalities

B1: CAO (accept equivalent inequalities but inequalities must not be strict)

Mark scheme (b)

Scheme	Marks	AO
$\left(\frac{2}{3}, \frac{5}{3}\right), \left(\frac{60}{37}, \frac{150}{37}\right), \left(\frac{25}{11}, \frac{36}{11}\right)$	B1	1.1b
	B1	1.1b

$\left(\frac{2}{3}, \frac{5}{3}\right) \rightarrow P = \frac{11}{3}$	M1	2.1
$\left(\frac{60}{37}, \frac{150}{37}\right) \rightarrow P = \frac{330}{37}$	A1	2.2a
$\left(\frac{25}{11}, \frac{36}{11}\right) \rightarrow P = \frac{111}{11}$ so optimal vertex is $\left(\frac{25}{11}, \frac{36}{11}\right)$		
	(4)	

Notes

(b) **B1:** One correct vertex (must be exact)

B1: All three correct vertices (must be exact)

M1: Testing all three of their vertices in the correct objective function

A1: Correct three values of P and correct optimal vertex either stated or clearly indicated on the graph

Mark scheme (c)

Scheme	Marks	AO
$Q = 3x + ay$		
$3\left(\frac{25}{11}\right) + \frac{36a}{11} > 3\left(\frac{60}{37}\right) + \frac{150a}{37}$	M1	3.1a
$\Rightarrow a < \frac{5}{2}$	A1	2.2a
$3\left(\frac{25}{11}\right) + \frac{36a}{11} > 3\left(\frac{2}{3}\right) + \frac{5a}{3}$	M1	1.1b
$\Rightarrow a > -3$	A1	2.2a
	(4)	
	(10 marks)	

Notes

(c) **M1:** Their optimal point from (b) evaluated in Q compared to their $\left(\frac{60}{37}, \frac{150}{37}\right)$ evaluated in Q (with correct inequality)

A1: $a < \frac{5}{2}$

M1: Their optimal point from (b) evaluated in Q compared to their $\left(\frac{2}{3}, \frac{5}{3}\right)$ evaluated in Q (with correct inequality)

A1: $a > -3$