

QQQ – Decision 1 – Chapter 3 – Algorithms on Graphs (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1.

	A	B	C	D	E	F
A	-	135	180	70	95	225
B	135	-	215	125	205	240
C	180	215	-	150	165	155
D	70	125	150	-	100	195
E	95	205	165	100	-	215
F	225	240	155	195	215	-

The table shows the lengths, in km, of potential rail routes between six towns, A, B, C, D, E and F.

- (a) Use Prim's algorithm, starting from A, to find a minimum spanning tree for this table. You must list the arcs that form your tree in the order that they are selected. **(3)**
- (b) Draw your tree using the vertices given in Diagram 1 in the answer book. **(1)**
- (c) State the total weight of your tree. **(1)**

(Total 5 marks)

2. The table below shows the lengths, in km, of the roads in a network connecting seven towns, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	24	–	22	35	–	–
B	24	–	25	27	–	–	–
C	–	25	–	33	31	36	26
D	22	27	33	–	–	42	–
E	35	–	31	–	–	37	29
F	–	–	36	42	37	–	40
G	–	–	26	–	29	40	–

- (a) By adding the arcs from vertex D along with their weights, complete the drawing of the network on Diagram 1 in the answer book. (2)

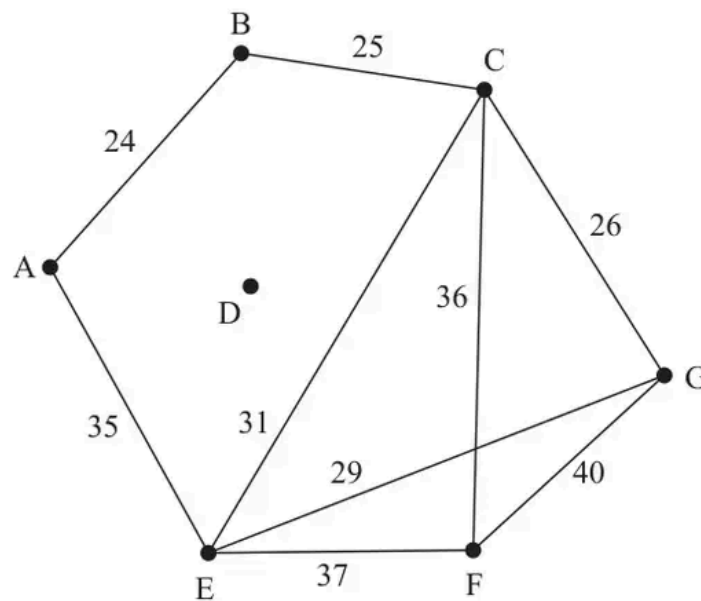


Diagram 1

- (b) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree. (3)
- (c) State the weight of the minimum spanning tree. (1)

(Total 6 marks)

3.

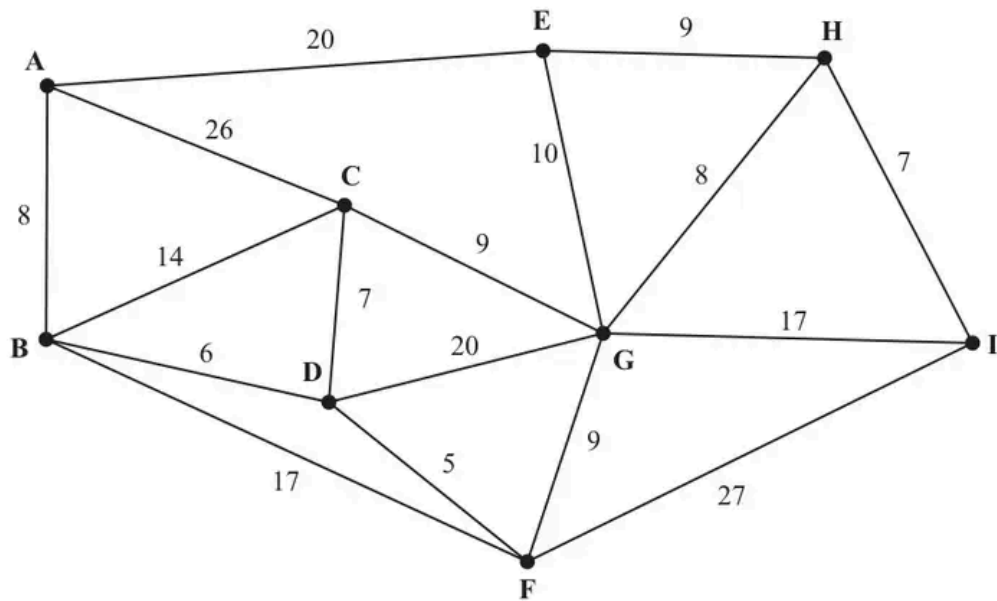


Figure 4

Figure 4 represents a network of roads. The number on each arc gives the length, in km, of that road.

- (a) Use Dijkstra's algorithm to find the shortest distance from A to I. State your shortest route. (6)
- (b) State the shortest distance from A to G. (1)

(Total 7 marks)

4.

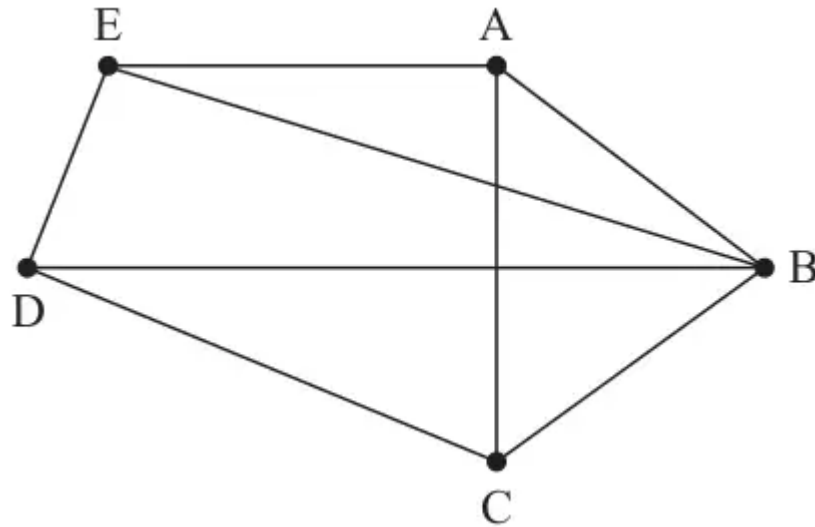


Figure 1

Figure 1 shows the graph G .

- (a) State whether G is Eulerian, semi-Eulerian, or neither, giving a reason for your answer. (1)
- (b) Write down an example of a Hamiltonian cycle on G . (1)
- (c) State whether or not G is planar, justifying your answer. (1)
- (d) State the number of arcs that would need to be added to G to make the graph K_5 (1)

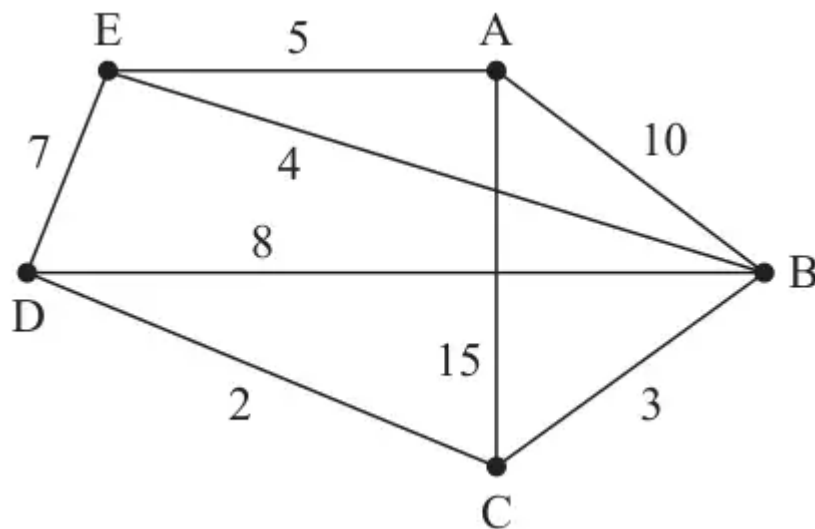


Figure 2

Direct roads between five villages, A, B, C, D and E, are represented in Figure 2. The weight on each arc is the time, in minutes, required to travel along the corresponding road. Floyd's algorithm is to be used to find the complete network of shortest times between the five villages.

- (e) For the network represented in Figure 2, complete the initial time matrix in the answer book. (1)

Initial time matrix (answer book)

	A	B	C	D	E
A	–				
B		–			
C			–		
D				–	
E					–

The time matrix after four iterations of Floyd's algorithm is shown in Table 1.

	A	B	C	D	E
A	–	10	13	15	5
B	10	–	3	5	4
C	13	3	–	2	7
D	15	5	2	–	7
E	5	4	7	7	–

Table 1

- (f) Perform the final iteration of Floyd's algorithm that follows from Table 1, showing the time matrix for this iteration. (2)

(Total 7 marks)

Solutions

1.

Mark scheme (a)

Scheme	Marks
AD, AE, DB; DC, CF	M1 A1; A1
	(3)

Notes

(a) 1M1: Using Prim – first 2 arcs probably but condone starting from another vertex.

1A1: first three arcs correct

2A1: all correct.

Apply the misread rule, if not listing arcs or not starting at A.

So for M1 (only)

Accept numbers across the top (condoning absence of 6)

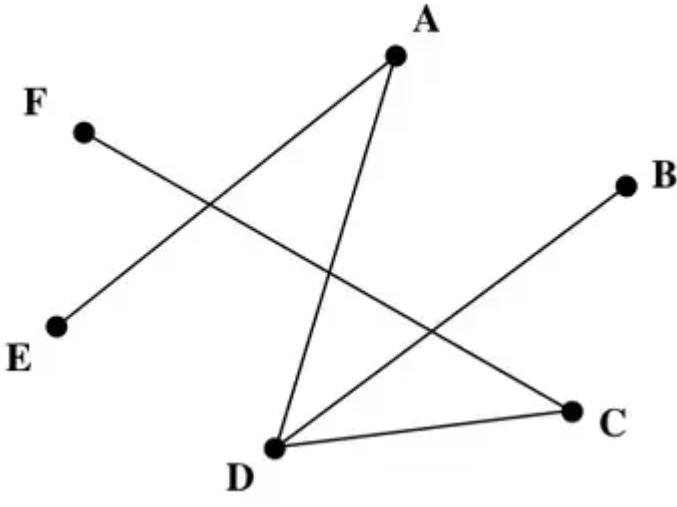
Accept full vertex listing

Accept full arc listing starting from vertex other than A

[AD AE DB DC CF]	{1 4 5 2 3 6}	ADEBCF
BD AD AE CD CF	{3 1 5 2 4 6}	BDAECF
CD AD AE BD CF	{3 5 1 2 4 6}	CDAEBF
DA AE DB CD CF	{2 4 5 1 3 6}	DAEBCF
EA AD DB DC CF	{2 4 5 3 1 6}	EADBCF
FC CD AD AE BD	{4 6 2 3 5 1}	FCDAEB

Mark scheme (b)

Scheme	Marks
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	B1
	(1)

Notes

(b) 1B1: CAO

Mark scheme (c)

Scheme	Marks
Weight 595 (km)	B1
	(1)
	(5 marks)

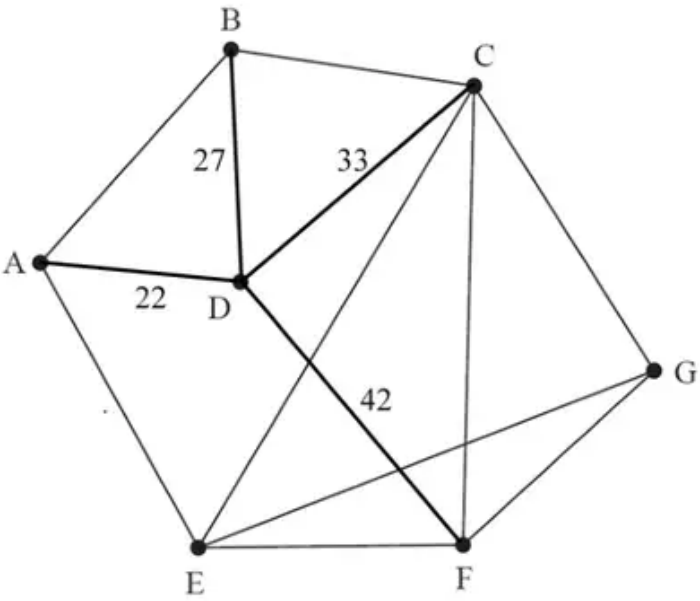
Notes

(c) 1B1: CAO condone lack of km.

2.

Mark scheme (a)

Scheme	Marks	AO
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	M1 A1	1.1b 1.1b
	(2)	

Notes

(a) **M1**: Either all arcs correct (ignore weights) or two arcs correct (including correct weights)

A1: CAO

Mark scheme (b)

Scheme	Marks	AO
Kruskal's algorithm: AD, AB, BC, CG, reject BD, EG, reject CE, reject CD, reject AE, CF (reject EF, reject FG, reject DF)	M1 A1 A1	1.1b 1.1b 1.1b
	(3)	

Notes

(b) **M1**: Kruskal's algorithm – first three arcs correctly chosen and at least one rejection seen at some point

A1: All six arcs selected correctly AD, AB, BC, CG, EG, CF only

A1: CSO – all selections and rejections correct (in correct order and at the correct time)

Mark scheme (c)

Scheme	Marks	AO
Weight of MST: 162 (km)	B1	1.1b

	(1)	
	(6 marks)	

Notes

(c) B1: CAO (condone lack of units)

3.

Mark scheme (a)

Scheme	Marks
	M1 1A1 2A1ft 3A1ft 4A1ft
Route: A E H I	5A1
	(6)

Notes

(a) 1M1: Small replacing big in the working values at C or F or G or I

1A1: Everything correct in boxes at A, B, D and F

2A1ft: ft boxes at E and C handled correctly but penalise order of labelling only once

3A1ft: ft boxes at G and H handled correctly but penalise order of labelling only once

4A1ft: ft boxes at I handled correctly but penalise order of labelling only once

5A1: route cao A E H I

Mark scheme (b)

Scheme	Marks
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Shortest distance from A to G is 28 km	B1ft
	(1)
	(7 marks)

Notes

(b) 1B1ft: ft their final label at G condone lack of km

4.

Mark scheme (a)

Scheme	Marks	AO
Graph G is neither as there are more than two vertices of odd degree	B1	2.4
	(1)	

Notes

B1: 'Neither' together with a correct reason (ignore irrelevant statements but do not isw incorrect statements)

Examples of B1:

- G contains more than 2 odd nodes
- G contains 4 odd nodes (**or** stating that A, C, D and E are odd)
- G contains (only) 1 even node
- The number of odd degree nodes in G is not 0 or 2

Examples of B0

- G contains odd nodes
- G contains at least 2 odd nodes
- G contains 4 nodes of degree 3 (not linking this to 'odd')

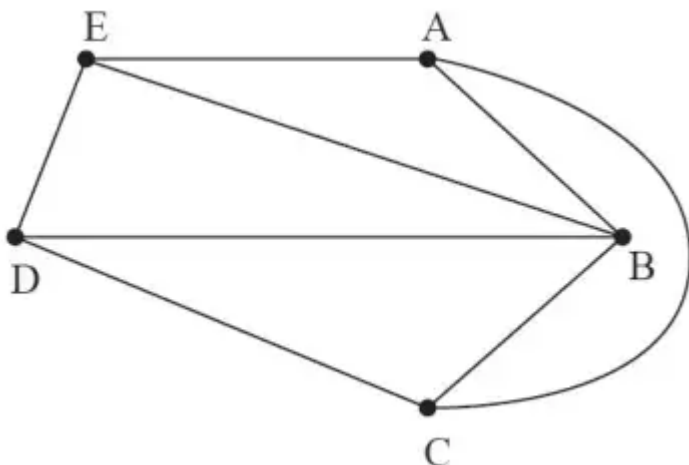
Mark scheme (b)

Scheme	Marks	AO
e.g. A – B – C – D – E – A	B1	1.1b
	(1)	

Notes

B1: CAO - must begin and end at the same node and include every other node in the graph exactly once.
Accept if given in terms of arcs e.g. AC, CB, BD, DE, EA

Mark scheme (c)

Scheme	Marks	AO
G <u>is</u> planar as it can be drawn with no arcs intersecting/crossing each other e.g. 	B1	2.4
	(1)	

Notes

B1: Correct answer of planar + justification

Examples of correct justification:

- AC can be moved outside **or** EB, DB can be moved outside
- AC(O), BE(I), BD(I) (or vice-versa) accept just AC(O) **or** BE(O) and BD(O)
- A correct drawing of the graph as planar (condone nodes not being labelled)

Examples of insufficient justification:

- Move arc BE (or BD) outside (need to mention both)
- Move arcs outside so that no arcs cross each other (must give specific examples of which arc(s) are being moved outside)
- Comments about moving nodes

Mark scheme (d)

Scheme	Marks	AO
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2	B1	1.1b
	(1)	

Notes

B1: CAO (2 only)

Mark scheme (e)

Scheme	Marks	AO																																				
<table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr><tr><td>A</td><td>-</td><td>10</td><td>15</td><td>∞</td><td>5</td></tr><tr><td>B</td><td>10</td><td>-</td><td>3</td><td>8</td><td>4</td></tr><tr><td>C</td><td>15</td><td>3</td><td>-</td><td>2</td><td>∞</td></tr><tr><td>D</td><td>∞</td><td>8</td><td>2</td><td>-</td><td>7</td></tr><tr><td>E</td><td>5</td><td>4</td><td>∞</td><td>7</td><td>-</td></tr></table>		A	B	C	D	E	A	-	10	15	∞	5	B	10	-	3	8	4	C	15	3	-	2	∞	D	∞	8	2	-	7	E	5	4	∞	7	-	B1	1.1b
	A	B	C	D	E																																	
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D	∞	8	2	-	7																																	
E	5	4	∞	7	-																																	
	(1)																																					

Notes

B1: CAO - no blanks entries (apart from the lead diagonal) and must include ∞ in cells AD, CE, DA, EC)

Mark scheme (f)

Scheme	Marks	AO																																				
<table><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr><tr><td>A</td><td>-</td><td>[9]</td><td>[12]</td><td>[12]</td><td>5</td></tr><tr><td>B</td><td>[9]</td><td>-</td><td>3</td><td>5</td><td>4</td></tr><tr><td>C</td><td>[12]</td><td>3</td><td>-</td><td>2</td><td>7</td></tr><tr><td>D</td><td>[12]</td><td>5</td><td>2</td><td>-</td><td>7</td></tr><tr><td>E</td><td>5</td><td>4</td><td>7</td><td>7</td><td>-</td></tr></table>		A	B	C	D	E	A	-	[9]	[12]	[12]	5	B	[9]	-	3	5	4	C	[12]	3	-	2	7	D	[12]	5	2	-	7	E	5	4	7	7	-	M1 A1	1.1b 1.1b
	A	B	C	D	E																																	
A	-	[9]	[12]	[12]	5																																	
B	[9]	-	3	5	4																																	
C	[12]	3	-	2	7																																	
D	[12]	5	2	-	7																																	
E	5	4	7	7	-																																	
	(2)																																					

	(7 marks)	
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Notes

M1: No change in the fifth row and fifth column with at least **two** values reduced correctly (no blank entries – apart from the lead diagonal)

A1: CAO for the final iteration