

# QQQ – Decision 1 – Chapter 2 – Graphs and Networks (Version 1)

**Total Marks: 25**

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

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1. Explain what is meant, in a network, by

- (a) a walk, (2)
- (b) a tour. (2)

**(Total 4 marks)**

Edexcel D2 June 2008 Q2

2.

- (a) Draw the graph  $K_5$  (1)
- (b) (i) In the context of graph theory explain what is meant by ‘semi-Eulerian’.
- (ii) Draw two semi-Eulerian subgraphs of  $K_5$ , each having five vertices but with a different number of edges. (3)
- (c) Explain why a graph with exactly five vertices with vertex orders 1, 2, 2, 3 and 4 cannot be a tree. (2)

**(Total 6 marks)**

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3.

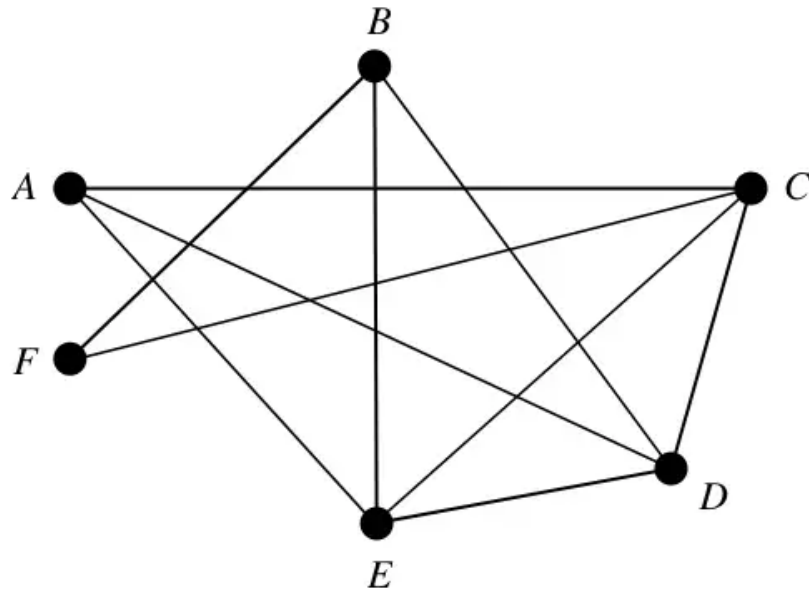


Figure 1

(a) Starting from  $A$ ; write down a Hamiltonian cycle for the graph in Figure 1. (2)

(b) Use the planarity algorithm to show that the graph in Figure 1 is planar. (3)

Arcs  $AF$  and  $EF$  are now added to the graph.

(c) Explain why the new graph is not planar. (2)

**(Total 7 marks)**

4.

- (a) Explain why it is **not** possible to draw a graph with exactly six nodes with degrees 1, 2, 3, 4, 5 and 6 (1)

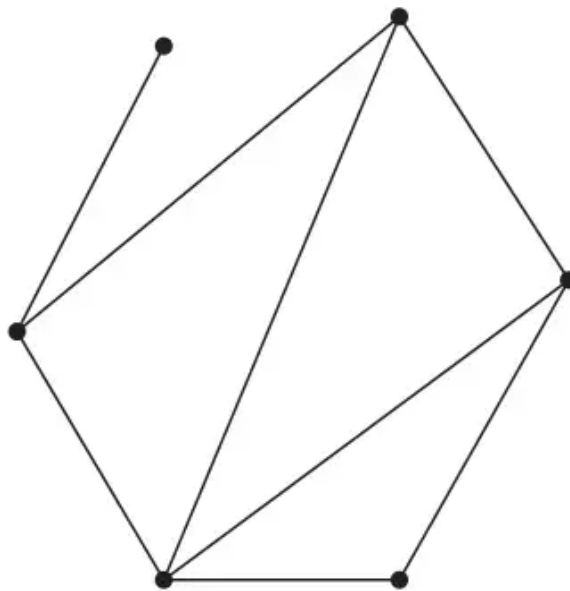
A tree,  $T$ , has exactly six nodes. The degrees of the six nodes of  $T$  are

$$1 \quad 2 \quad (4-x) \quad (2x-5) \quad (4x-11) \quad (3x-5)$$

where  $x$  is an integer.

- (b) Explain how you know that  $T$  cannot be Eulerian. (1)

- (c) (i) Determine the value of  $x$
- (ii) Hence state whether  $T$  is semi-Eulerian or not. You must justify your answer. (5)



### Figure 2

Figure 2 shows a graph,  $G$ , with six nodes with degrees 1, 2, 3, 3, 3 and 4

- (d) Using the vertices in Diagram 1 in the answer book, draw a graph with exactly six nodes with degrees 1, 2, 3, 3, 3 and 4 that is **not** isomorphic to G. (1)



**Diagram 1**

**(Total 8 marks)**

Edexcel A2 June 2024 Q4

# Solutions

1.

## Mark scheme (a)

| Scheme                                                                                                                  | Marks    |
|-------------------------------------------------------------------------------------------------------------------------|----------|
| A walk is a <b>finite sequence of arcs</b> such that the <b>end vertex of one arc is the start vertex of the next</b> . | B2, 1, 0 |
|                                                                                                                         | (2)      |

### Notes

1B1: Probably one of the two below but accept correct relevant statement – bod gets B1, generous.

2B1: A good clear complete answer: End vertex = start vertex + finite.

## Mark scheme (b)

| Scheme                                                                               | Marks     |
|--------------------------------------------------------------------------------------|-----------|
| A tour is a walk that visits <b>every vertex, returning to its starting vertex</b> . | B2, 1, 0  |
|                                                                                      | (2)       |
|                                                                                      | (4 marks) |

### Notes

1B1: Probably one of the two below but accept correct relevant statement – bod gets B1, generous.

2B1: A good clear complete answer: Every vertex + return to start.

### From the D1 and D2 glossaries

#### D1

A **path** is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, and in which no vertex appears more than once.

A **cycle (circuit)** is a closed path, ie the end vertex of the last edge is the start vertex of the first edge.

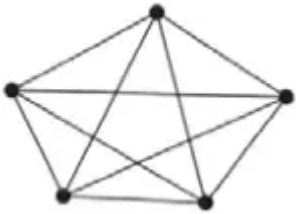
#### D2

A **walk** in a network is a finite sequence of edges such that the end vertex of one edge is the start vertex of the next.

A walk which visits every vertex, returning to its starting vertex, is called a **tour**.

2.

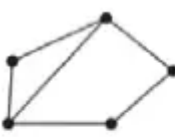
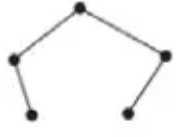
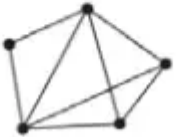
Mark scheme (a)

| Scheme                                                                            | Marks | AO  |
|-----------------------------------------------------------------------------------|-------|-----|
|  | B1    | 1.2 |
|                                                                                   | (1)   |     |

Notes

**B1:** CAO (give bod for position of nodes)

Mark scheme (b)

| Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Marks    | AO           |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|--------------|
| (i) A semi-Eulerian graph contains <u>exactly two nodes of odd order</u> (and any number of nodes of even order)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | B1       | 2.5          |
| (ii) e.g. (two semi-Eulerian subgraphs of $K_5$ with a different number of edges) <div style="display: flex; flex-wrap: wrap; justify-content: space-around; margin-top: 20px;">    </div> <div style="display: flex; flex-wrap: wrap; justify-content: space-around; margin-top: 20px;">    </div> | B1<br>B1 | 1.1b<br>1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | (3)      |              |

Notes

**(b)(i) B1:** CAO (accept 'there are exactly two odd nodes' but must contain exact oe (e.g. 'only two odd nodes' or 'all but 2 nodes have an even order' but not 'the graph has two odd nodes'))

**(b)(ii) B1:** One correct semi-Eulerian subgraph of  $K_5$  with five nodes

**B1:** Two correct semi-Eulerian subgraphs of  $K_5$  with five nodes – note that the graphs must have a different number of edges

### Mark scheme (c)

| Scheme                                                                                                                             | Marks       | AO          |
|------------------------------------------------------------------------------------------------------------------------------------|-------------|-------------|
| e.g. The graph with five vertices has $\frac{1 + 2 + 2 + 3 + 4}{2} = 6$ arcs<br>but a tree on five nodes would contain only 4 arcs | B1<br>B1dep | 2.2a<br>2.4 |
|                                                                                                                                    | (2)         |             |
|                                                                                                                                    | (6 marks)   |             |

### Notes

**B1:** Deducing that the graph has 6 arcs **or** a tree on five nodes has 4 arcs **or** the node of order 4 must be connected to the other 4 nodes **or** an argument based on the sum of the orders of both the graph and the tree (but must relate the orders to the number of arcs and not the number of nodes) **or** the node with order 4 and one of the nodes of orders 2 or 3 would create a cycle **or** a tree must have two nodes of order 1

**B1dep:** Complete argument – graph has 6 arcs and the tree would only have 4 arcs **or** the sum of the orders is 12 compared to 8 for the tree **or** the node of order 4 must be connected to the other 4 nodes therefore all the other vertices would have to have order 1 **or** the graph has 6 arcs and therefore with 5 vertices there would have to be cycles **or** the node of order 4 is connected to the other 4 nodes and so together with the node of order 3 (or 2) a cycle would be formed **or** a tree must have at least two nodes of order 1 as otherwise a cycle would be formed

Note: no marks in (c) for attempts based only on examples of graphs drawn with the vertex orders as stated

## 3.

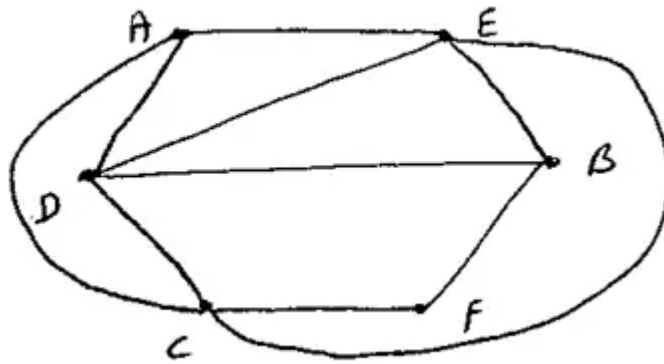
### Mark scheme (a)

| Scheme              | Marks |
|---------------------|-------|
| e.g. <i>AEBFCDA</i> | M1 A1 |
|                     | (2)   |

### Mark scheme (b)

| Scheme | Marks |
|--------|-------|
|--------|-------|

e.g.



M1 A1 A1

(3)

#### Mark scheme (c)

| Scheme                                                                                                    | Marks     |
|-----------------------------------------------------------------------------------------------------------|-----------|
| State that one of these arcs ( $AF$ or $EF$ ) [Named], crosses at least one arc in each set. [Named arcs] | B2,1,0    |
|                                                                                                           | (2)       |
|                                                                                                           | (7 marks) |

4.

#### Mark scheme (a)

| Scheme                                                                                                                                                                                                        | Marks | AO  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-----|
| e.g. A graph cannot contain an odd number of odd vertices<br>e.g. $\frac{1 + 2 + 3 + 4 + 5 + 6}{2} = 10.5$ which is not an integer and so therefore not possible to have a graph with the given vertex orders | B1    | 1.2 |
|                                                                                                                                                                                                               | (1)   |     |

#### Notes

(a) In a and b condone poor language such as number of odd degrees instead of number of odd nodes

B1: CAO – common examples that score B1:

- Cannot have (a graph with an) **odd number of odd vertices**

- **Cannot** have a graph with **three odd vertices**
- The **sum of the degrees/order** (of the vertices) **is 21** which is **not even** therefore **not possible** (but not just for obtaining 21 and saying 'impossible'). The 21 must be linked either in words to the 'sum of the degrees/order' **or** explicitly showing  $1 + 2 + 3 + 4 + 5 + 6 = 21$  so just '21 is not even' scores B0
- The **sum of the degrees/order** (of the vertices) **is 21** which is **odd** therefore **not possible** (with equivalent justification of the 21 as in the previous bullet-point)
- $\frac{1 + 2 + 3 + 4 + 5 + 6}{2} = 10.5$  which is **not an integer** so therefore **impossible**. They do not have to explain that they are using the result that  $\sum \text{vertex degrees} = 2(\text{no of arcs})$  but they must explain why a value of 10.5 leads to the required graph not being possible. A value of 10.5 with no working (or explanation) scores B0

### Mark scheme (b)

| Scheme                                                                    | Marks      | AO  |
|---------------------------------------------------------------------------|------------|-----|
| e.g.<br>T has at least one node of degree one or one node with odd degree | B1         | 2.4 |
|                                                                           | <b>(1)</b> |     |

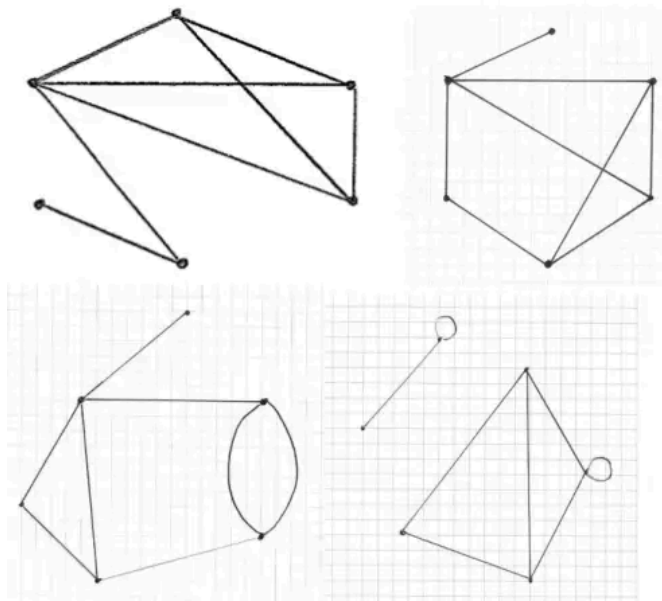
#### Notes

**(b) B1:** CAO – correct reasoning that not all the vertices of T can have an even degree. Allow for this mark a general statement that no tree can be Eulerian as all trees contain at least two nodes of degree 1. Allow as a minimum statement that at least one of the nodes has a degree of 1 or at least one odd node (accept valency instead of degree or order)

### Mark scheme (c)

| Scheme                                                                                                                                                                                                  | Marks | AO   |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 6 nodes in T therefore the tree contains 5 arcs                                                                                                                                                         | B1    | 1.2  |
| $1 + 2 + (4 - x) + (2x - 5) + (4x - 11) + (3x - 5) = 2(5)$<br>$(8x - 14 = 10)$                                                                                                                          | M1    | 3.1a |
| $x = 3$                                                                                                                                                                                                 | A1    | 1.1b |
| Alternatively<br>$4 - x > 0 \Rightarrow x < 4$ $4 - x \geq 1 \Rightarrow x \leq 3$<br>$2x - 5 > 0 \Rightarrow x > 2.5$ $4x - 11 \geq 1 \Rightarrow x \geq 3$<br>$2.5 < x < 4 \Rightarrow x = 3$ $x = 3$ |       |      |





(1)

(8 marks)

### Notes

**B1:** CAO (any graph that is not isomorphic to  $G$  – must contain exactly 8 arcs) If a simple, connected graph the vertex with degree 1 must connect to either the vertex of degree 2 or degree 4. Note this does **not** have to be a simple graph and may **not** be connected

(Degrees are 1, 2, 3, 3, 3, 4)