

QQQ – Decision 1 – Chapter 1 – Algorithms (Version 2)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 15 = Bronze)

1.

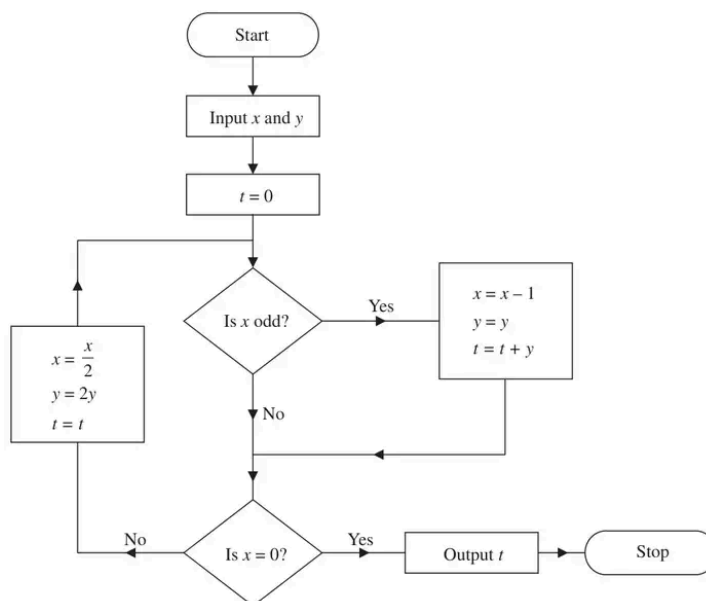


Figure 4

An algorithm is described by the flow chart shown in Figure 4.

Given that $x = 27$ and $y = 5$,

- (a) complete the table in the answer book to show the results obtained at each step when the algorithm is applied. Give the final output. **(4)**

The numbers 122 and $\frac{1}{2}$ are to be used as inputs for the algorithm described by the flow chart.

- (b) (i) State, giving a reason, which number should be input as x .
(ii) State the output. **(3)**

(Total 7 marks)

2. The nine distinct numbers in the following list are to be packed into bins of size 50

23	17	19	x	24	8	18	10	21
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When the first-fit bin packing algorithm is applied to the numbers in the list it results in the following allocation.

Bin 1: 23 17 8

Bin 2: 19 x 10

Bin 3: 24 18

Bin 4: 21

(a) Explain why $13 < x < 21$ (3)

The same list of numbers is to be sorted into descending order. A bubble sort, starting at the left-hand end of the list, is to be used to obtain the sorted list. After the first complete pass the list is

23	19	17	24	x	18	10	21	8
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(b) Using this information, write down the smallest interval that must contain x , giving your answer as an inequality. (2)

When the first-fit decreasing bin packing algorithm is applied to the nine distinct numbers it results in the following allocation.

Bin 1: 24 23

Bin 2: 21 19 10

Bin 3: 18 17 x

Bin 4: 8

Given that only one of the bins is full and that x is an integer,

(c) calculate the value of x . You must give reasons for your answer. (2)

(Total 7 marks)

3.

2.1	1.7	3.0	1.9	3.2	1.2	3.3	1.4	1.5	0.2
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- (a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 5 (2)

The list of numbers is now to be sorted into descending order.

- (b) Perform a quick sort on the original list to obtain the sorted list. You should show the result of each pass and identify your pivots clearly. (4)

For a list of n numbers, the quick sort algorithm has, on average, order $n \log n$.

Given that it takes 2.32 seconds to run the algorithm when $n = 450$

- (c) calculate approximately how long it will take, to the nearest tenth of a second, to run the algorithm when $n = 11\,250$. You should make your method and working clear. (2)

(Total 8 marks)

Solutions

1.

Mark scheme (a)

Scheme						Marks
x	y	t	Is x odd?	Is $x = 0$?		M1 (3 rows + 1 st correct) A1 (2 nd and 3 rd rows correct) A1 (4 th , 5 th and 6 th rows correct)
27	5	0	Yes			
26	(5)	5		No	Row 1	
13	10	(5)	Yes		Row 2	
12	(10)	15		No	Row 3	
6	20	(15)	No	No	Row 4	
3	40	(15)	Yes		Row 5	
2	(40)	55		No	Row 6	
1	80	(55)	Yes			
0	(80)	135		Yes		
		(135)				
Output = 135						A1 (CSO)
						(4)

Notes

Candidates may write each changed value/statement in a new row which is fine. Assume that each row begins and ends when a value in x is changed. For example, the values in row 1 in the table above consists of the x values going from the 26 to the 13

a1M1: At least three rows of cells in columns x , y and t completed with a correct first row (so 26 for x and 5 for t)

a1A1: CAO – second and third rows correct (for just the columns in x , y and t)

a2A1: CAO – fourth, fifth and sixth rows correct (for just the columns in x , y and t)

a3A1: CSO – including the output of 135 **either** on the given line in the answer book **or clearly** stated in the table but it **must** be absolutely clear that the output is the final t value (no bod). Furthermore, all ‘yes’

and 'no' comments must be present in the 4th and 5th columns with no additional/incorrect 'yes' or 'no'

Mark scheme (b)

Scheme	Marks
(i) x must be a (positive) integer and therefore $x = 122$	B1 DB1
(ii) 61	B1
	(3)
	(7 marks)

Notes

bi1B1: x must be 122 and **any** attempt at a reason

bi2DB1: Dependent on previous B mark (so B0B1 is not possible) – 122 and a correct valid reason – e.g. x must be an integer/whole number **or** $\frac{1}{2}$ is not odd or even **or** if you input $\frac{1}{2}$ then you can never get to $x = 0$ when halving, etc. Just saying that the algorithm 'won't work' or that the algorithm 'will get stuck in a loop' or 'not terminate' is not sufficient for this second mark neither is the argument of subtracting 1 from a $\frac{1}{2}$. It must be clear **why** the algorithm won't output a value for t with $x = \frac{1}{2}$ - so essentially there needs to be some indication of why x will never become 0. Furthermore, just saying that x will never reach 0 is insufficient – we need an indication of **why** $x = 0$ is not possible with a starting value of $x = \frac{1}{2}$

bii3B1: CAO

2.

Mark scheme (a)

Scheme	Marks	AO
If x has been placed in Bin 2 then $10 < x \leq 31$ - this is because Bin 1 at this stage only contains 40 and before x had been placed in Bin 2 it only contained 19		
As the 18 has been placed in Bin 3 this implies that $x > 50 - (19 + 18)$ so $x > 13$	B1	3.1a
As the 10 has been placed in Bin 2 after the x then $x \leq 50 - (19 + 10)$ so $x \leq 21$	B1	2.4
However, the number are all distinct and therefore $13 < x < 21$	B1	2.2a

	(3)	
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Notes

(a) **B1:** Correct reasoning of why $x > 13$ accept $x > 50 - (19 + 18)$

B1: Correct explanation of why $x \leq 21$ accept $x \leq 50 - (19 + 10)$

B1: Correct deduction that $13 < x < 21$ must mention that the numbers are distinct (oe)

Mark scheme (b)

Scheme	Marks	AO
$13 < x < 17$	B1	2.2a
	B1	2.2a
	(2)	

Notes

(b) **B1:** Use first complete pass to deduce that $x < 17$

B1: Correct lower bound of $x > 13$

Mark scheme (c)

Scheme	Marks	AO
If x has been placed in Bin 3 then this implies that $x \leq 15$	M1	2.4
So x is either 14 or 15 – but as Bin 2 is full $\Rightarrow x = 14$	A1	2.2a
	(2)	
	(7 marks)	

Notes

(c) **M1:** Using first-fit decreasing in an attempt to derive new upper bound for x (so either for stating both x could equal 14 or 15, $x \leq 15$ or $x < 15$)

A1: Correct deduction that $x = 14$ (must clearly state or imply that Bin 2 is full)

3.

Mark scheme (a)

Scheme	Marks	AO
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										3 . 2 (0 . 2)	
3 . 3	<u>3.2</u>	2.1	1.7	3.0	1.9	1.4	1.5	<u>1.2</u>	0 . 2	P i v o t (s) : (3 . 3) 1 . 9	
3 . 3	<u>3.2</u>	2.1	3.0	<u>1.9</u>	1.7	1.4	1.5	<u>1.2</u>	0 . 2	P i v o t s : 3	

										· 0		
										1 · 4		
3 · 3	<u>3.2</u>	<u>3.0</u>	2.1	<u>1.9</u>	1.7	1.5	<u>1.4</u>	<u>1.2</u>	0 · 2	P i v o t (s) : (2 · 1) 1 · 5		
3 · 3	<u>3.2</u>	<u>3.0</u>	2.1	<u>1.9</u>	1.7	<u>1.5</u>	<u>1.4</u>	<u>1.2</u>	0 · 2			
											(4)	

Notes

(b) M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration then max of M1A1 only** – Bubble sort is not a MR and scores M0

A1: First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct) – so they must be choosing (if middle right) a pivot value of 3.2 for the second pass or

(if middle left) a pivot value of 1.9

A1ft: Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark

A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including if middle right a fifth pass with the 1.5 used as a pivot or if middle left a fourth pass with the 1.7 used as a pivot

Sorting list into ascending order in (b)

- If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in **(b)**
- If the list is not reversed in **(b)** then remove the last two A marks earned in **(b)**. If the candidate says that the list needs reversing in **(b)** but does not actually show the reversed list in **(b)** then remove the last A mark earned
- **Note that if sorting into ascending order then a 'sort complete' statement is required – this could be shown by the final list being re-written or 'sorted' statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice) BEFORE list is reversed**

Middle left

2.1	1.7	3.0	1.9	<u>3.2</u>	1.2	3.3	1.4	1.5	0.2
3.3	<u>3.2</u>	2.1	1.7	3.0	<u>1.9</u>	1.2	1.4	1.5	0.2
3.3	<u>3.2</u>	<u>2.1</u>	3.0	<u>1.9</u>	1.7	1.2	<u>1.4</u>	1.5	0.2
3.3	<u>3.2</u>	3.0	<u>2.1</u>	<u>1.9</u>	<u>1.7</u>	1.5	<u>1.4</u>	<u>1.2</u>	0.2
3.3	<u>3.2</u>	3.0	<u>2.1</u>	<u>1.9</u>	<u>1.7</u>	1.5	<u>1.4</u>	<u>1.2</u>	0.2

Middle right ascending (which requires a 'sort complete' statement – see above)

2.1	1.7	3.0	1.9	3.2	<u>1.2</u>	3.3	1.4	1.5	0.2
0.2	<u>1.2</u>	2.1	1.7	3.0	1.9	<u>3.2</u>	3.3	1.4	1.5
0.2	<u>1.2</u>	2.1	1.7	3.0	<u>1.9</u>	1.4	1.5	<u>3.2</u>	3.3
0.2	<u>1.2</u>	1.7	<u>1.4</u>	1.5	<u>1.9</u>	2.1	<u>3.0</u>	<u>3.2</u>	3.3
0.2	<u>1.2</u>	<u>1.4</u>	1.7	<u>1.5</u>	<u>1.9</u>	2.1	<u>3.0</u>	<u>3.2</u>	3.3
0.2	<u>1.2</u>	<u>1.4</u>	<u>1.5</u>	1.7	<u>1.9</u>	2.1	<u>3.0</u>	<u>3.2</u>	3.3

Middle left ascending (which required a 'sort complete' statement – see above)

2.1	1.7	3.0	1.9	<u>3.2</u>	1.2	3.3	1.4	1.5	0.2
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2.1	1.7	3.0	<u>1.9</u>	1.2	1.4	1.5	0.2	<u>3.2</u>	3.3
1.7	1.2	<u>1.4</u>	1.5	0.2	<u>1.9</u>	<u>2.1</u>	3.0	<u>3.2</u>	3.3
<u>1.2</u>	0.2	<u>1.4</u>	<u>1.7</u>	1.5	<u>1.9</u>	<u>2.1</u>	3.0	<u>3.2</u>	3.3
0.2	<u>1.2</u>	<u>1.4</u>	1.5	<u>1.7</u>	<u>1.9</u>	<u>2.1</u>	3.0	<u>3.2</u>	3.3

Mark scheme (c)

Scheme	Marks	AO
$\frac{2.32 (11\,250 \log 11\,250)}{450 \log 450}$	M1	1.1a
= 88.6 seconds	A1	1.1b
	(2)	
	(8 marks)	

Notes

(c) **M1**: Complete correct method – allow reciprocal – allow slips in values only e.g. 1250 for 11 250

A1: CAO – the exact value of 88.6 must be stated at some point (as question specifically asked for the answer to the nearest tenth of a second) – isw if 90 follows 88.6 seen. 90 with no working scores no marks.

An answer of 88.6 with no working scores M1A0 – condone lack of units (but if present must be correct)