

QQQ – Core Pure 2 – Chapter 8 – Modelling with Differential Equations (Version 2)

Total Marks: 24

(24 = Platinum, 22 = Gold, 19 = Silver, 16 = Bronze)

1. The movement of a particle is described by the simple harmonic equation

$$\ddot{x} = -25x$$

where x metres is the displacement of the particle at time t seconds, and $\ddot{x} \text{ m s}^{-2}$ is the acceleration of the particle.

The maximum displacement of the particle is 9 metres.

Find the maximum speed of the particle.

Circle your answer.

[1 mark]

- 15 m s^{-1}
- 45 m s^{-1}
- 75 m s^{-1}
- 135 m s^{-1}

(Total 1 marks)

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2. The solution of a second order differential equation is $f(t)$

The differential equation models heavy damping.

Which one of the statements below could be true?

Tick (✓) **one** box.

[1 mark]

- $f(t) = 2e^{-t} \cos(3t) + 5e^{-t} \sin(3t)$
- $f(t) = 3e^{-t} + 4te^{-t}$
- $f(t) = 7e^{-t} + 2e^{-2t}$
- $f(t) = 8e^{-t} \cos(3t - 0.1)$

(Total 1 marks)

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3. Two different colours of paint are being mixed together in a container.

The paint is stirred continuously so that each colour is instantly dispersed evenly throughout the container.

Initially the container holds a mixture of 10 litres of red paint and 20 litres of blue paint.

The colour of the paint mixture is now altered by

- adding red paint to the container at a rate of 2 litres per second
- adding blue paint to the container at a rate of 1 litre per second
- pumping fully mixed paint from the container at a rate of 3 litres per second.

Let r litres be the amount of red paint in the container at time t seconds after the colour of the paint mixture starts to be altered.

- (a)** Show that the amount of red paint in the container can be modelled by the differential equation

$$\frac{dr}{dt} = 2 - \frac{r}{\alpha}$$

where α is a positive constant to be determined. **(2)**

- (b)** By solving the differential equation, determine how long it will take for the mixture of paint in the container to consist of equal amounts of red paint and blue paint, according to the model. Give your answer to the nearest second. **(6)**

It actually takes 9 seconds for the mixture of paint in the container to consist of equal amounts of red paint and blue paint.

- (c)** Use this information to evaluate the model, giving a reason for your answer. **(1)**

(Total 9 marks)

4. A biologist is studying the effect of pesticides on crops. On a certain farm pesticide is regularly applied to a particular crop which grows in soil. Over time, pesticide is transferred between the crop and the soil at a rate which depends on the amount of pesticide in both the crop and the soil. The amount of pesticide in the crop after t days is x grams. The amount of pesticide in the soil after t days is y grams. Initially, when $t = 0$, there is no pesticide in either the crop or the soil.

At first it is assumed that no pesticide is lost from the system. The biologist further assumes that pesticide is added to the crop at a constant rate of k grams per day, where $k > 6$.

After collecting some initial data, the biologist suggests that for $t \geq 0$, this situation can be modelled by the following pair of first order linear differential equations.

$$\frac{dx}{dt} = -2x + 78y + k$$

$$\frac{dy}{dt} = 2x - 78y$$

(a) (i) Show that $\frac{d^2x}{dt^2} + 80\frac{dx}{dt} = 78k$. [2]

(ii) Determine the particular solution for x in terms of k and t . [7]

If more than 250 grams of pesticide is found in the crop, then it will fail food safety standards.

(iii) The crop is tested 50 days after the pesticide is first added to it.

Explain why, according to this model, the crop will fail food safety standards as a result of this test. [1]

Further data collection suggests that some pesticide decays in the soil and so is lost from the system. The model is refined in light of this data. The particular solution for x for this refined model is

$$x = k \left(20 - e^{-41t} \left(20 \cosh(\sqrt{1677} t) + \frac{819}{\sqrt{1677}} \sinh(\sqrt{1677} t) \right) \right).$$

(b) Given now that $k < 12$, determine whether the crop will fail food safety standards in the long run according to this refined model. [2]

In the refined model, it is still assumed that pesticide is added to the crop at a constant rate.

(c) Suggest a reason why it might be more realistic to model the addition of pesticide as not being at a constant rate. [1]

(Total 13 marks)

