

QQQ – Core Pure 2 – Chapter 8 – Modelling with Differential Equations (Version 1)

Total Marks: 25

(25 = Platinum, 23 = Gold, 20 = Silver, 17 = Bronze)

1. The displacement of a door from its equilibrium (closed) position is measured by the angle, θ radians, which the door makes with its closed position. The door can swing either side of the equilibrium position so that θ can take positive and negative values. The door is released from rest from an open position at time $t = 0$.

A proposed differential equation to model the motion of the door for $t \geq 0$ is

$$\frac{d^2\theta}{dt^2} + \lambda \frac{d\theta}{dt} + 3\theta = 0 \text{ where } \lambda \text{ is a constant and } \lambda \geq 0.$$

- (a) (i) According to the model, for what value of λ will the motion of the door be simple harmonic? [1]
- (ii) Explain briefly why modelling the motion of the door as simple harmonic is unlikely to be realistic. [1]
- (b) Find the range of values of λ for which the model predicts that the door will never pass through the equilibrium position. [2]
- (c) Sketch a possible graph of θ against t when λ lies **outside** the range found in part (b) but the motion is not simple harmonic. [1]

(Total 5 marks)

2. A raindrop falls from rest from a cloud. The velocity, $v \text{ m s}^{-1}$ vertically downwards, of the raindrop, t seconds after the raindrop starts to fall, is modelled by the differential equation

$$(t + 2) \frac{dv}{dt} + 3v = k(t + 2) - 3 \quad t \geq 0$$

where k is a positive constant.

(a) Solve the differential equation to show that

$$v = \frac{k}{4}(t + 2) - 1 + \frac{4(2 - k)}{(t + 2)^3}$$

(5)

Given that $v = 4$ when $t = 2$

(b) determine, according to the model, the velocity of the raindrop 5 seconds after it starts to fall. **(3)**

(c) Comment on the validity of the model for very large values of t **(1)**

(Total 9 marks)

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3. Solve the simultaneous differential equations

$$\frac{dx}{dt} + 2x = 4y, \quad \frac{dy}{dt} + 3x = 5y,$$

given that when $t = 0$, $x = 0$ and $y = 1$.

[11]

(Total 11 marks)

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